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A Better Approach to Goodness: Reply to Wagemans (1999)

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In P. A. van der Helm and E. L. J. Leeuwenberg (1996), the authors presented a representation model for the goodness, or detectability, of visual regularities such as mirror symmetry and repetition. J. Wagemans (1999) acknowledged that this *holographic* goodness model has considerable explanatory power, but he also argued that it is not good enough yet. He challenged van der Helm and Leeuwenberg to qualify some open ends of their representation model, in particular those concerning its process assumptions. He also questioned the authors' assessment of previous goodness accounts such as S. E. Palmer's (1982, 1983) *transformational approach* and his own *bootstrap model.* He concluded that it is expedient to aim at a synthesis of useful aspects of diverse accounts of goodness, but he did not establish such a synthesis. Van der Helm and Leeuwenberg agree with his conclusion that such a synthesis is a worthy cause, but they disagree with his evaluation of the issues involved. This article is a reply with an alternative evaluation of these issues, advancing the discussion to a process–representation synthesis called *holographic bootstrapping.*

Goodness of visual regularity refers to the detectability of single regularities and combinations of regularities, whether or not perturbed by noise. In Van der Helm and Leeuwenberg (1996) we proposed a model to quantify goodness. Wagemans (1999) presented a critical comment on our goodness model. Before addressing Wagemans's comment, we first sketch how our goodness model combines the principles of the *holographic approach* (Van der Helm & Leeuwenberg, 1991) with the encoding principles of Leeuwenberg's (1969, 1971) structural information theory (SIT).

The holographic approach comprises a strictly mathematical formalization of regularity in symbol sequences. The core of this formalization is the specification of the internal structure a regularity should have to be perceptually relevant. In simple terms, the structure should be such that all of its substructures reflect the same kind of regularity. The formalization resulted in pinpointing the unique status of only three regularities: repetition, bilateral symmetry, and so-called alternation. By *repetition* we mean juxtaposed identical subpatterns. By *bilateral symmetry* we not only mean mirror symmetry but also broken symmetry (in this reply, we confine ourselves to mirror symmetry). An instance of alternation is given by the randomly positioned but coherently oriented dot pairs (*dipoles*) in Glass patterns. At present, these three regularities are precisely the ones that are considered in SIT. For instance, the repetition of *abc* in the symbol sequence *abc abc* can be encoded in SIT as 2*(*abc*). In general, the parentheses in a SIT-code specify the parts involved in holographic identity relationships (*identities*). Thus, the SIT-code 2*(*abc*) describes one holographic identity, namely, the identity between the two subsequences *abc.* Another instance is the mirror symmetry in the symbol sequence *abc cba,* which can be encoded in SIT as S[(*a*)(*b*)(*c*)]. This code describes three holographic identities, namely, the identities between the symbols *a*, the symbols *b*, and the symbols *c*.

In our holographic goodness model, we proposed $W = E/n$ as a goodness measure. W refers to weight of evidence, quantified by the number of holographic identities described by a SIT-code (E), divided by the number of elements in the symbol sequence (n). In the repetition example above, $E = 1$ and $n = 6$, so that the repetition gets a goodness value of $W = 1/6$. This illustrates that the goodness value W of a repetition depends on the total number of elements in the symbol sequence. In contrast, the goodness value of a mirror symmetry is always $W = 1/2$. This is illustrated by the mirror symmetry example just described, in which $E = 3$ and $n = 6$, so that the mirror symmetry gets a goodness value of $W = 3/6 = 1/2$.

To allow our goodness model to be applied to 2-D patterns, we generalized the principles mentioned earlier. The essence of this generalization is that, just as in 1-D symbol sequences, 2-D repetition has a block structure consisting of a small number of identities between relatively large pattern parts (the repeated parts), whereas 2-D mirror symmetry has a point structure consisting of a large number of identities between *pixels* (basic pattern elements). This structural difference between repetition and mirror symmetry explains the well-known phenomenon that mirror symmetry is "better" than repetition, and it is the starting point for the explanation of many other known goodness phenomena (for details, see our original article: Van der Helm & Leeuwenberg, 1996). We also predicted new phenomena. For instance, we predicted that noisy mirror symmetries comply with Weber's Law which, as we argued, explains a *symmetry effect,* that is, a perceptual bias toward symmetry (see, e.g., Carmody, Nodine, & Locher, 1977; Freyd & Tversky, 1984). In addition, we predicted that threefold mirror symmetry is "worse" than twofold mirror symmetry. This counterintuitive prediction has recently gained some empirical support from Wenderoth and Welsh (1998).

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In his constructive comment, Wagemans (1999) acknowledged that our holographic goodness model has considerable explanatory power, but he also argued that it is not good enough yet. His conclusion was that better results might be obtained by an account that combines useful aspects of process accounts such as his own bootstrap model (Wagemans, Van Gool, Swinnen, & Van Horebeek, 1993) with useful aspects of representation accounts such as Palmer's (1982, 1983) transformational approach and our holographic approach. He did not establish such a synthesis but did identify and evaluate several obstacles that seem to obstruct such a synthesis.

We agree with Wagemans that such a synthesis is a worthy cause, but we disagree with his evaluation of the theoretical issues involved. In fact, we see no real obstacles for such a synthesis. We argue that the holographic approach can bridge the gap between process and representation accounts, advancing the discussion to a process–representation synthesis, which we call *holographic bootstrapping.* To this end, we first reply to Wagemans's comments on our evolutionary considerations, our assessment of the transformational approach, and our choice to develop a representation model before a process model. Then we discuss the issues of proximity and salient subpatterns, which Wagemans emphasized as being major obstacles for a synthesis. Finally, we present our synthesis proposal of holographic bootstrapping.

Evolution, Growth, and Holographic Regularity

In our original article, we argued that many process theorists adhere to the idea that the perceptual sensitivity to certain regularities has been caused by their abundance in nature during the evolution. Wagemans argued that the cause is not abundance but biological significance. As we sketch next, however, abundance and significance are neither separable nor conclusive causes.

First, the visual system obviously has to be able to process a sufficient diversity of biologically significant information, otherwise it simply would not survive. This, however, does not necessarily imply that the visual system processes biologically significant information only. Hence, not every internal aspect of perception necessarily requires a justification in terms of the external world. Second, in perception the biological significance of mirror symmetry is often based on the idea that it is characteristic for objects. Indeed, it is characteristic for objects, precisely because symmetrical forms occur so abundantly in nature. Moreover, in biology the biological significance of mirror symmetry is generally based on the alternative idea that symmetry perturbations reflect growth perturbations, so that degree of symmetry indicates quality. This is supposed to have led, in animals, to a preference for symmetrical forms to mate with or to pollinate (Møller, 1995) so that, as a consequence, symmetrical forms occur so abundantly in nature. This symmetry preference, however, could just as well be a consequence of high perceptual sensitivity to mirror symmetry (Enquist & Arak, 1994). Thus, the perceptual sensitivity may indirectly have caused the abundance of symmetry, so that symmetry automatically became characteristic for objects. In contrast to Wagemans, we adhere to the latter idea: We think that the perceptual sensitivity is the primary cause. We also think that the fact that quality is indicated by degrees of symmetry concurs with the perceptual sensitivity because the perceptual sensitivity, too, is based on the underlying principle of growth. This principle, for perception, is sketched next.

The growth of most symmetrical forms in nature occurs by a gradual addition of fragments, preserving the global symmetrical form. This is also the essence of the holographic criterion for the intrinsic character of visual regularity. In the holographic approach, ordered sets of identities (*identity chains*) are the formal tools to describe any arbitrary case of regularity. Such an identity chain is holographic if, to put it simply, each of its subchains describes the same kind of regularity. A holographic identity chain belongs to what mathematicians call a *unary algebra* (cf. Birkhoff & Bartee, 1970). This means that it can be uniquely expanded, one identity at a time, preserving its holographic character (i.e., the identity chain can be expanded to another identity chain from the same unary algebra). This reflects the aspect of growth. For instance, all cases of bilateral symmetry in symbol sequences together form such a unary algebra of holographic identity chains. Therefore, bilateral symmetry is said to be a *holographic regularity.* The formal criterion then states that only holographic regularities qualify as visual regularities. Only 20 holographic regularities have been shown to exist. By means of the encoding criterion of *transparent hierarchy* (which we do not elaborate on in this article), this set is further narrowed down to repetition, bilateral symmetry, and alternation. In sum, the holographic approach provides a formal criterion for visual regularity based on the evolutionary principle of growth.

Position of the Transformational Approach

The transformational approach (Palmer, 1982, 1983) provides an alternative underlying principle plus a formal criterion for visual regularity. Before we go into Wagemans's comments on our assessment of this approach, we first discuss it briefly. In the transformational approach, transformations are the formal tools to describe any arbitrary case of regularity. These tools do not differ essentially from the holographic tools. Some tools are just more convenient than others to formulate a specific criterion. The transformational approach only provides a formal criterion for visual regularity if it confines itself to so-called *rigid* whole-pattern transformations to assess pattern regularity. (The mathematically powerful concept of *groups* is often focused on in the transformational approach; however, as we argued in our original article, it does not seem to be a very useful factor in characterizing visual regularity; cf. Cutting, 1987.) The focus on rigid whole-pattern transformations reflects the criterion that regularities are perceptually relevant only if they exhibit invariance under 3-D rotations and translations. Mirror symmetry and repetition exhibit such invariance. The rigidity criterion, in turn, reflects that motion can be seen as the underlying principle. Wagemans reproached us for insisting on mathematical rigor in our assessment of the transformational approach. We did so, however, to be able to pinpoint the rigidity criterion and the motion principle. Without them, the transformational approach would not provide a formal foundation for visual regularity, as we argue next.

Whereas the holographic approach implies a block structure for repetition and a point structure for mirror symmetry, the transformational approach implies a block structure for both regularities. For instance, for mirror symmetry the so-called *flip* and *reflection* both are transformations that treat the two symmetry halves as single-unit blocks. (Unlike what Wagemans suggested, we do interpret the reflection as operating on a whole pattern.) Wagemans acknowledged that the transformational block structure of mirror symmetry is perceptually implausible, but he also argued that this flaw may be "fixed" by allowing what he calls the *local TA*. The local TA is a set of nonrigid pattern transformations, each of which interchanges a symmetry pair consisting of two transformationally related parts; that is, two parts that are each other's mirror image, such as ◀▷ and ◁▶. Unlike Wagemans's suggestion, the local TA is not the same as the set Φ of nonrigid pattern transformations specified in our original article. Each Φ-transformation interchanges a symmetry pair consisting of two identical parts. Moreover, for nonrigid transformations, no criterion is known to specify only a restricted set of perceptually relevant regularities (cf. Van der Helm, 1994). Hence, if one allows the local TA or Φ, one creates a win–lose situation: One wins perceptual plausibility, because the local TA and Φ describe mirror symmetry as having a point structure, but one loses a formal foundation. In contrast, the holographic approach creates a win–win situation: It provides for the desired point structure of mirror symmetry as well as for a formal foundation—a much better fix!

The foregoing indicates that, by specifying the choice of the regularities to be processed, the transformational and holographic approaches are formal theories that take up what one could call a metamodel position. Neither theory comprises a ready-made process or representation model, but each theory does impose qualitative restrictions on both types of models. To be compatible with a formal theory, such a model should implement the theoretically specified regularities as well as the theoretically specified structure of each of those regularities. For instance, SIT's encoding model is a representation model compatible with the holographic approach. Furthermore, it is true that Palmer (1982, 1983) proposed a fairly plausible process model based on so-called *local spatial analyzers* (LSAs) but, unlike Wagemans's suggestion, this LSA model is not compatible with the transformational approach, as we discuss next.

At best, Palmer's LSA model can be said to be transformationally inspired. Whereas the transformational approach implies a block structure for both repetition and mirror symmetry, the LSA model uses a point structure for both regularities. The LSA model starts from pairs of small, transformationally related receptive fields. For repetition, an LSA pair responds to identical input. For mirror symmetry, an LSA pair responds to mutually mirror-reversed input (similar to the symmetry pairs in Wagemans' local TA, except that in the local TA a pair is not confined to receptive fields). The LSA model could describe a twofold repetition or mirror symmetry as having a block structure, but only if the two pattern halves fall precisely in two transformationally related receptive fields. In general, however, the LSA pairs, in fact, positionally correspond to the endpoints of the so-called *virtual lines* in Wagemans's bootstrap model. In the bootstrap model, two virtual lines may form a mirror-symmetric trapezoid (in the case of mirror symmetry) or a parallelogram (in the case of repetition). These so-called *correlation quadrangles* are the starting points from which the bootstrap process propagates. Whether, for mirror symmetry, the endpoints of a virtual line reflect mutually mirror-reversed input (as in the LSA model) or identical input (as in the bootstrap model) is not relevant for the bootstrap mechanisms. Hence, the bootstrap model can be seen as a concrete implementation of Palmer's conceptually described LSA model. As we argue later on, the bootstrap model (and thereby the LSA model) can be modified such that it is compatible with the holographic approach.

Representation Model Before Process Model

Wagemans questioned our choice of developing a representation model instead of a process model. He argued that the process has some logical priority over representations because the process precedes the representations. However, the fact that causes precede consequences does not mean that one should study the "first" cause first. To humans, consequences are primary and, although Wagemans disagreed, even in evolution consequences can be said to be primary. The biotope of an individual organism is insensitive to the ways this organism extracts information from its environment; the organism is simply killed if it does not get sufficiently adequate information. This means that there may be room for a gradual improvement of the extraction process, but only if the primary condition (i.e., the extraction of sufficiently adequate information) has been satisfied. Furthermore, in science one starts from observed phenomena, and one searches backward for causes. For instance, in experimental psychology the goal is to perform experiments in such a way that the responses exhibit a certain systematicity. This systematicity then calls for an explanation and, in the search backward for its causes, the first candidate cause encountered is the representation. This is why we prefer to search for an explanation in terms of representation systematicity first. Having found such an explanation, we are, of course, not exempted from searching further backward for compatible process systematicity. It is fortunate that much research on process models is already being performed by others; we might benefit from this.

We agree with Wagemans that proposed process and representation models cannot simply be combined because we, too, see several obstacles that seem to obstruct such a synthesis. For instance, process models such as the bootstrap model relate experimental responses to dynamic properties of the extraction process, whereas static properties of the resulting representation are not supposed to play a role. For representation models, such as our holographic model, it is precisely the other way around. How can these seemingly contradictory explanatory principles be reconciled? Furthermore, the bootstrap model extracts repetition in a pointwise fashion. How can this be reconciled with the seemingly contradictory holographic block structure of repetition? These obstacles, however, will be overcome by our synthesis proposal of holographic bootstrapping.

Finally, some clarifying statements are in order in view of Wagemans's comments that SIT adopts implausible process assumptions. In fact, SIT actually makes no explicit assumptions at all about the internal mechanisms of the extraction process. SIT only specifies the character of representations, that is, the outcome of the extraction process. Of course, this means that SIT implicitly imposes qualitative constraints on the process. In short, the process should be such that it results in the simplest hierarchical representation from which the stimulus can be reconstructed (for details, see our original article). Unlike Wagemans's suggestion, SIT's process constraints do not imply that the process should result in symbol sequences as used in SIT. It seems expedient to clarify this further, even though in our original article we used SIT's symbol sequences merely as an analogy to illustrate the holographic principles applied to 2-D patterns.

First, in the 1960s and 1970s, SIT was developed as an experimenter's tool to investigate preferred pattern interpretations. SIT's raw representation of a stimulus, by means of a symbol sequence, was left to be established by the experimenter in a psychologically acceptable way, given the type of stimuli used in the experiment at hand. At that time, SIT was recognized as such, and the main process problem was the seemingly unrealistic selection of a simplest code among the combinatorially explosive number of codes for a given symbol sequence (cf. Hatfield & Epstein, 1985). However, this process problem has since been solved. Using the principles of the holographic approach, the encoding algorithm PISA computes a guaranteed simplest code for any given symbol sequence without combinatorial explosions (Van der Helm, 1988; Van der Helm & Desain, 1999; Van der Helm & Leeuwenberg, 1986, 1991).

Second, SIT's symbol sequences are merely references to perceptual input. For instance, a symbol might refer to the input of a receptive field, just as the word *chair* is a symbol referring to a physical entity. Such a reference is not a direct reference, but it is mediated by the mental concept of the physical entity (cf. Lyons, 1977). In fact, it is the mental concept that psychological research is after, and the usage of references is a means in this research. In this sense, both process and representation models are reference systems. SIT's symbolism is typical for representation models. It enables research into (e.g., hierarchical) relations between perceptual entities without having to specify precisely how those entities are processed. A process model such as the bootstrap model uses another terminology (i.e., other symbols) that enables research into the processing of perceptual entities without being specific about how they are represented. For instance, Wagemans (1999) specifies a representation as "simply a pattern of activation," (p. 617) which is at least as scanty as SIT's process assumptions. This difference between process and representation models is not so much an obstacle as a gap between two scientific paradigms. We do not pretend to close this gap, but our synthesis proposal of holographic bootstrapping will bridge this gap (starting from our representation model). We will first address the issues of proximity and salient subpatterns which Wagemans emphasizes as being major obstacles for such a synthesis.

Effect of Proximity

Wagemans suggested that we underestimate the role of proximity in the extraction process. We indeed conceive proximity as a metrical aspect, which our approach does not account for explicitly. Nevertheless, our approach is still consistent with effects of proximity. For instance, during the extraction process, the holographic point structure of mirror symmetry allows proximity to play an additional role (e.g., near the symmetry axis), whereas the holographic block structure of repetition does not. Unlike what Wagemans suggested, we therefore do not see a contradiction in the implementation of heuristics reflecting proximity effects or, similarly, orientation effects, in a process model. As far as we are concerned, such heuristics may be added for the purpose of getting an improved simulation. The point is, however, that we prefer to deal with metrical and viewpoint-dependent aspects of perception in a more integrated way, that is, without taking refuge in separate heuristics each time a perceptual bias is discovered (see, e.g., Van Lier, Van der Helm, & Leeuwenberg, 1994). We conceive such heuristics as mere simulations of perceptual biases that, if possible, should get a more satisfactory explanation than just a reference to, for example, neuroanatomy. For instance, we think that the proximity effect is not merely a matter of (relative) metrical distances but rather an integral part of a more structural factor. Two candidates for this structural factor are *uniform connectedness* (Palmer & Rock, 1994) and *spatial contiguity* (Van der Helm & Leeuwenberg, 1996), as we discuss next.

The principle of uniform connectedness states that a closed region of homogeneous properties (e.g., color) tends to be organized initially as a single perceptual unit. For instance, in a dot pattern of nonoverlapping dots, each dot constitutes such a uniformly connected region and is, therefore, initially perceived as a single perceptual unit. If the pattern contains, for example, overlapping dots or dots connected by a line, then such an ensemble is initially perceived as a single perceptual unit. In a similar manner, a "blob," as produced by a spatial filter at a specific scale, initially constitutes a single perceptual unit at that specific scale. Palmer and Rock (1994) argued that the initial organization of the visual field into such units reflects the entry level for further organizing processes, allowing both parsing of these units into subordinate parts and grouping into superordinate structures (for related empirical work, see, e.g., Van Lier & Wagemans, 1998; Saiki & Hummel, 1998).

Spatial contiguity is somewhat more general than uniform connectedness and is an integral part of the holographic approach. A spatially contiguous region in a symbol sequence is simply a subsequence, that is, a set of successive symbols and not a set of symbols randomly drawn from all over the sequence. Furthermore, the substructures of a holographic structure in a symbol sequence are always disjunct subsequences, that is, disjunct, spatially contiguous regions in the symbol sequence. This property can be generalized straightforwardly to 2-D patterns by taking the substructures of a holographic structure in a 2-D pattern to be disjunct, spatially contiguous regions in this 2-D pattern. This means, in simple terms, that the 2-D pattern can be broken into a mosaic of pieces such that each of the substructures of the holographic structure corresponds precisely to one of these pieces. For instance, the holographic block structure of repetition is consistent with the fact that the juxtaposed copies in a repetition are disjunct, spatially contiguous regions. In contrast, the two slightly shifted, superimposed copies in a Glass pattern generally are not disjunct, spatially contiguous regions, so that they cannot be substructures in the holographic description of a Glass pattern. Instead, the holographic description of a Glass pattern consists of many substructures corresponding to the randomly positioned but coherently oriented dipoles that generally do form disjunct, spatially contiguous regions. This illustrates our holographic view that twofold repetitions and Glass patterns are two structurally different regularities, which contrasts with Wagemans's view that they have the same structure. Furthermore, if the dipole length in a Glass pattern is gradually increased, then fewer dipoles will form disjunct, spatially contiguous regions, which is reflected in a gradual deterioration of the detectability of the Glass pattern (a case of noise; cf. Maloney, Mitchison, & Barlow, 1987). Wagemans attributed this deterioration to proximity per se. In that case, however, one would also expect a deterioration for twofold repetition when an empty space is inserted between the two juxtaposed halves, whereas this actually yields an improvement (Corballis, Miller, & Morgan, 1971; Corballis & Roldan, 1974).

In summary, the preceding arguments suggest that the organizing process initially locks onto units corresponding to uniformly connected regions, whereas the resulting organization is formed by units corresponding to spatially contiguous regions. Two more distant points in a pattern will, on average, have less chance of both belonging to one such unit. Thus, such units can perhaps be approximated by implementing a process heuristic favoring proximity per se, but the intended effect is actually an integral part of structural factors, as alluded to by concepts such as uniform connectedness and spatial contiguity (cf. Palmer & Rock, 1994). Both concepts still need further elaboration, but we believe they will prove to be useful in perception research. In any case, we do not seem to need proximity per se as an independent factor in our goodness model. The holographic structure of visual regularity already seems to offer sufficient explanatory power.

The Effect of Salient Subpatterns in Mirror Symmetry

In our original article, we imposed a few plotting restrictions on dot patterns (e.g., nonoverlapping dots) to ensure that the dots were indeed the perceptually basic elements (Van der Helm & Leeuwenberg, 1996). This allowed us to keep our quantitative exposition relatively simple, and it prevented the primary subject of investigation (i.e., regularity) from being confounded by other factors. Wagemans suggested that we might have imposed these restrictions to avoid problems with perceptually salient subpatterns (e.g., due to proximity). Although this was not the case, the topic of salient subpatterns does have relevance in our discussion because we do not agree with Wagemans's argument that salient subpatterns always improve the goodness of a global mirror symmetry.

If subpatterns are salient because of extra regularity (e.g., collinearity), both the bootstrap model and the holographic model predict that they indeed improve goodness. (Both models may still have some technical problems with certain types of extra regularity but, in principle, they agree on this prediction.) Empirical support for this comes from, among many others, Locher and Wagemans (1993), who used mirror-symmetric stimuli built up from clusters of basic elements. We gather from this study that all of the clusters used were identical and that, moreover, each cluster itself was mirror symmetric. Wagemans used this study to question the holographic model because of the fact that, at larger spatial scales, these clusters form separate blobs (see later), but has he considered the extra regularity within and between the clusters?

If subpatterns are salient because of proximity only (so that, at larger spatial scales, one gets blobs), then our model does not predict goodness improvement. This is in agreement with Wenderoth's (1996) empirical data. So far, the bootstrap model complies with this as well. Yet, Wagemans favored an extension of the bootstrap model to include facilitation by blobs. He argued that a blob may become a preclustered single unit from which bootstrapping may start. To sustain this, Wagemans referred to Labonté, Shapira, Cohen, and Faubert (1995) and Wenderoth (1995) who, however, did not focus on the effect of salient subpatterns inside mirror symmetry. They focused on the effect of the degree to which other perceptual factors separate an entire mirror symmetry from its context. Wagemans also referred to Dakin and Watt (1994) who, however, actually concluded that "human data [on mirror symmetry detection] match the performance of a fairly fine-scale filter" (p. 411). In sum, the empirical evidence is against rather than in favor of an extension of the bootstrap model to include facilitation by blobs.

Extension of the bootstrap model to include facilitation by blobs also creates several theoretical problems. We agree with Palmer and Rock (1994) that blobs may be single entry-level units that may, however, also be parsed into subordinate parts. This seems plausible; after all, a blob is generally not shapeless. This implies that some kind of blob effect might occur; that is, the shape of a blob might be more easily parsed than the same region at a smaller scale. However, this is not what Wagemans meant. He wanted to consider blobs as units with a position only, that is, without shape. This would mean, however, that a mirror-symmetric blob can no longer be assessed as being mirror symmetric. A mirror symmetry often contains several such blobs aligned along the axis of symmetry. Wagemans argued that such an alignment of blobs (taken as single shapeless units) can be used to detect mirror symmetry. However, what about perfectly aligned but asymmetric blobs? In other words, blob alignment can be used to get a first indication that a symmetry axis might be present, but it is not sufficient (cf. Dakin & Watt, 1994). Each of the aligned blobs has to be parsed as well, either as a blob-with-shape, or as a region at a smaller scale. Besides, the aligned blobs cannot serve as single units from which bootstrapping starts: They cannot be endpoints of virtual lines spanning the (potential) symmetry axis indicated by the alignment.

If one half of a mirror symmetry contains a blob, then the other symmetry half contains the mirror image of that blob. As mentioned earlier, Wagemans argued that these two blobs might constitute single elements yielding a virtual line from which the bootstrapping process starts searching for mirror symmetry. But this begs the question because it still implies that the process has to be able to preassess that the two blobs are each other's mirror image. More or less analogous to Palmer's (1982, 1983) LSA model, Wagemans suggested that this preassessment may be performed in the spirit of the earlier mentioned local TA. However, Palmer's LSA model is still plausible in that it confines such a preassessment to transformationally related receptive fields, whereas Wagemans allowed the preassessment for entire blobs. Why not allow the preassessment for regions that might be entire symmetry halves so that bootstrapping becomes superfluous? Furthermore, how is the preassessment performed? In contrast to what Wagemans suggested, we do not think that the answer can be found in some kind of auto- or cross-correlation mechanism. First, such a mechanism does not properly reflect human regularity detection—it works too well (Barlow & Reeves, 1979; Dakin & Watt, 1994; Tapiovaara, 1990). Second, even though a correlation mechanism may operate essentially differently from a bootstrapping mechanism, it is also a parsing mechanism, and having two mechanisms do a similar job would not be parsimonious.

The bootstrap model, as it stands, considers virtual lines between identical elements, which agrees with the holographic approach—let's keep it this way. Much of the attraction of the present bootstrap model depends on the fact that its mechanisms can be used not only to assess global regularity but also to determine whether two pattern regions are identical or mirror-reversed by parsing these regions into smaller parts, down to the level of receptive fields. Blobs are perceptually relevant, and they might have a facilitating effect on repetition (see next section) but probably not on mirror symmetry.

Synthesis: Holographic Bootstrapping

In our original article, we were not very enthusiastic about the pointwise fashion in which the bootstrap model deals with repetition, that is, by means of correlation parallelograms. This seemed to contradict the holographic block structure of repetition, but we did not have an alternative process proposal either. Challenged by Wagemans's comments, however, we found an alternative within the framework of his bootstrap mechanisms, yet perfectly in line with our holographic approach, thus establishing a synthesis. With respect to the goodness differences between mirror symmetry and repetition, our proposal does not rely on, for example, the number of propagation directions or proximity, as does that of Wagemans. Instead, it relies on structural aspects of the building up of regularities. We illustrate this first in terms of the SIT-encoding of symbol sequences.

First, the mirror symmetry in the symbol sequence *abcdeffedcba* can, in part, be described by the SIT-code $ab\ S[(c)(d), (effe)]\ ba$. This SIT-code only describes the symmetry relations for the elements *c* and *d*. In bootstrap terms, this corresponds to a single correlation trapezoid. This partial description of the mirror symmetry can be expanded to, for instance, the SIT-code $a\ S[(b)(c)(d)(e), (ff)]\ a$. This corresponds, in bootstrap terms, to the inclusion of additional correlation trapezoids. Thus, the holographic growth of the SIT-code corresponds with the way Wagemans's bootstrap model proceeds in the search for mirror symmetry.

Second, in our original article we showed that Glass patterns can be encoded by the regularity called *alternation.* In general, alternation can be seen as a case of noisy or incomplete repetition. A typical example of alternation is the repeated occurrence of *abc* in the symbol sequence *abc p abc qr abc s,* which can be described by the SIT-code $< (abc) > / < (p)(qr)(s) >$. Thus, inversely, repetition can be seen as a kind of limit case of alternation. This is illustrated next.

The symbol sequence *abx abcde abcde abz* consists of four subsequences all beginning with *ab,* whereas the second and third subsequences are identical. Both aspects are, in part, described by the SIT-code $abx < (ab) > / < (cde)(cde) > abz$. This SIT-code only describes the identity of the second and third occurrences of *ab.* In bootstrap terms, this corresponds to a single correlation parallelogram. Holographically, this partial description may grow, or may be expanded, in two ways (just as one correlation parallelogram may indicate either a translational Glass pattern or a repetition). First, the two identical clusters *ab* may induce a search for additional clusters *ab,* yielding, for example, the SIT-code $< (ab) > / < (x)(cde)(cde)(z) >$. Here, one may conceive the clusters *ab* as referring to the dipoles in a Glass pattern, and the rest as referring to the dipole positions. Second, the two identical clusters *ab* may also induce a search for expansion of these two clusters themselves, yielding, for example, the SIT-code $abx < (abc) > / < (de)(de) > abz$. This expansion may eventually result in the description of the repetition of *abcde.* Although, in bootstrap terms, both expansions imply that correlation parallelograms are combined, the SIT-codes show that the two expansions are structurally different from each other.

In Wagemans's bootstrap model, Glass patterns and repetitions are dealt with in exactly the same way; that is, the structural difference between these two regularities is not revealed. The foregoing SIT analogy in terms of 1-D symbol sequences, however, indicates how this structural difference can be translated into 2-D bootstrap mechanisms. In short, bootstrapping should proceed in agreement with the way representations grow holographically. For mirror symmetry and translational Glass patterns, this holographic bootstrapping hardly differs from Wagemans's bootstrap model, but for repetition the difference is fundamental (see Figure 1). As we show in this section, this difference has far-reaching implications. In general, the automatic search process starts from randomly established virtual lines and quadrangles. Although bootstrapping allows parallel propagation from different starting points, we describe and evaluate propagation from only one point.

Suppose that a correlation trapezoid has been found as an indication of the presence of mirror symmetry (Figure 1A). Then, for both virtual lines, the process searches in parallel for new "partners" (other virtual lines) to form additional correlation trapezoids. This search is repeated, simultaneously for all virtual lines found so far, thus propagating very fast. The lightly shaded areas in Figure 1A indicate two symmetrically related, spatially contiguous regions that together form a subsymmetry. The partner search has to preserve this spatial contiguity. This condition is more easily satisfied by partner dots nearby (proximity effect). It also implies that noise dots may break a global mirror symmetry into several separate subsymmetries (in line with our representational account). The spatial contiguity condition has similar implications for repetition and for Glass patterns, but we do not elaborate on this here.

When a correlation parallelogram has been found, it may indicate the presence of either a translational Glass pattern or a twofold repetition. In the Glass pattern search (Figure 1B), the four dots of the parallelogram are clustered into two dipoles along the (supposed) translation vector. For both dipoles, the process subsequently searches in parallel for new partners, that is, other identical dipoles, which means that the already clustered dipoles have to be parsed. The dipoles and their new partners form additional correlation parallelograms. The search is repeated, simultaneously for all dipoles found so far, thus propagating relatively fast.

In the repetition search (Figure 1C), the four dots of the parallelogram are also clustered into two single units but, this time, not along the (supposed) translation vector. These two identical blocks constitute one new virtual line, for which the process searches a partner (another virtual line) to form anew a correlation parallelogram. If found, the new dots are included to form two larger identical blocks, and so on, obviously propagating much more slowly than in the case of a Glass pattern. This shows that the holographic structuring implies different propagation strategies for repetition and Glass patterns, replicating their goodness difference much more convincingly than Wagemans's bootstrap model. Moreover, unlike Wagemans's bootstrap model, holographic bootstrapping has the following qualitative implications.

For repetition, the propagation spreads linearly, each step adding one correlation parallelogram. For mirror symmetry, the propagation spreads exponentially, each step adding more and more correlation trapezoids in parallel. This is in agreement with Baylis and Driver's (1994) conclusion that repetition is dealt with in a

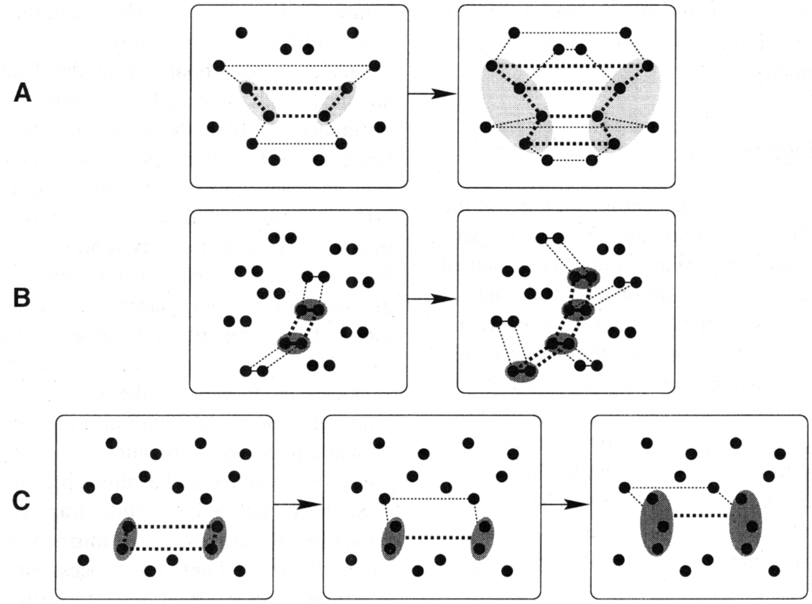

Figure 1. Holographic bootstrapping in (A) a mirror symmetry, (B) a translational Glass pattern, and (C) a repetition. The bold, dashed lines indicate correlation quadrangles. The thin, dashed lines indicate the search for additional correlational quadrangles. The shaded areas indicate spatially contiguous regions (not necessarily ellipse shaped). The lightly shaded areas in A indicate that the search has to preserve this spatial contiguity. The same holds for B and C, but here such lightly shaded areas have been left out. Instead, the darkly shaded areas in B and C indicate clustering into single spatially contiguous units.

serial fashion and mirror symmetry in a parallel fashion. The linear propagation in repetition implies that variation in the number of dots strongly affects the detection time (in line with our representational account). In contrast, the exponential propagation in mirror symmetry and Glass patterns implies that, except for small numbers of dots, variation in the number of dots hardly affects the detection time (approximately in line with our representational account; see also Tapiovaara, 1990). For instance, in mirror symmetry, propagation spreads over 35 dots just as fast as over 60 dots. If the number of dots is small (e.g., fewer than 20), Glass patterns seem to be less easily detected than mirror symmetry (in line with our representational account), probably because of the required parsing of the dipoles. Furthermore, in general, extra local regularity supplies additional correlations that might be exploited to speed up the partner search. Mirror symmetry benefits less from this than repetition, because in mirror symmetry the propagation already progresses very fast so that extra regularity has relatively little beneficial effect on speed. This reflects a kind of ceiling effect which, because of the coding principle of transparent hierarchy, is inherently present in our representational account.

To conclude our evaluation of holographic bootstrapping, we return to the issue of salient subpatterns. The following analysis is perhaps somewhat complex, but the result is very intriguing. As mentioned earlier, subpatterns that are salient because of proximity or local regularity might be detected and clustered before a global regularity is detected. Then, the question arises, Which effect would preclustering have? Suppose, first, that two such subpatterns (one in each half of a global mirror symmetry or repetition) are not preclustered and that the global regularity is processed as usual, starting from points inside these subpatterns. Assume that this would take a total detection time of $T_{pattern}$. Now suppose that, temporarily disregarding the global regularity, only these subpatterns are preprocessed and preclustered—taking a time of T_{subs}. Subsequently, the rest of the global regularity is processed starting from the correlations resulting from the preprocess—taking a time of T_{rest}. Then, in repetition, the total time $T_{subs} + T_{rest}$ is about equal to $T_{pattern}$, whereas in mirror symmetry it is much larger than $T_{pattern}$. In other words, it is true that in both repetition and mirror symmetry, the rest of the global regularity benefits from preprocessing the subpatterns. However, the total repetition benefits fully from the preclustering, whereas the total mirror symmetry is better off without the preclustering! In holographic bootstrap terms, this is clarified as follows.

In repetition, the preprocess clusters the subpatterns little by little into two identical single units, just as would be the case without preclustering. These two units form a virtual line from which the rest of the repetition can be processed, as if the subpatterns were two basic elements in the repetition. In mirror symmetry, however, by the time the usual process (without preclustering) has processed the subpatterns, it has in fact already processed a much larger region because of the exponential spreading of the propagation. Hence, in fact, preclustering works counterproductively with respect to the detection of the global mirror symmetry! This analysis does not specify when preclustering occurs, but it does describe the effect in case preclustering does occur. This effect is in line with the concept of transparent hierarchy (see our original article) and can, for instance, be obtained by inserting an empty space between the two pattern halves of a twofold repetition or a mirror symmetry. In this case, each pattern half behaves like a separate cluster, which strengthens the repetition but weakens the mirror symmetry (Baylis & Driver, 1995; Corballis et al., 1971; Corballis & Roldan, 1974; Kahn & Foster, 1986; see our original article, Van der Helm & Leeuwenberg, 1996, p. 453).

Conclusion

In this reply to Wagemans (1999), we clarified our metatheoretical views to sketch the line of research that resulted in the holographic model of the goodness, or detectability, of visual regularities (Van der Helm & Leeuwenberg, 1996). We argued that the holographic approach (Van der Helm & Leeuwenberg, 1991) is a formal theory that takes up a metamodel position; that is, it is primarily concerned with the choice of the regularities to be processed. Thereby, it imposes qualitative restrictions on perceptual models concerned with the extraction and representation of visual regularity. Leeuwenberg's (1969, 1971) structural information theory (SIT) provides a representation model compatible with the holographic approach. Our holographic goodness model combines the principles of the holographic approach with SIT's encoding principles.

We also argued that the bootstrap model (Wagemans et al., 1993) can be seen as an implementation of Palmer's (1982, 1983) transformationally inspired LSA model. We showed that the bootstrap model (and thereby the LSA model) can be modified regarding repetition detection, such that it is compatible with the holographic approach. To replicate the various goodness behaviors of visual regularities, our holographic bootstrap model does not rely on proximity, for example, but instead on the structural organization of these regularities. Much better than Wagemans's bootstrap model, our holographic bootstrap model explains, in a qualitative sense, a variety of goodness phenomena, just as our representational goodness model already did in a quantitative sense. Therefore, the synthesis of a holographic process model (using bootstrap mechanisms) and a holographic representation model (using SIT's structural encoding) yields a scientifically more satisfactory and more comprehensive approach to the goodness of visual regularities.

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