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A code-theoretical note on object handedness

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Abstract. This study is a theoretical exercise dealing with discrimination between images and mirror-images. It focuses on the way codes of shapes represent their handedness. We compare two code systems with different reference frames. These frames determine the specific sensitivity of each system. One system uses an asymmetric reference frame. It is called the H-system and was inspired by an idea of Corballis (1988 *Psychological Review* **95** 115 – 123). The other system, being our proposal, uses a symmetric reference frame and we have named it the M-system. We demonstrate the following. A code of the H-system provides a cue for the handedness of a shape, but not for rotation, ie no cue for the appropriate kind of code rotation which should be tested in case images and mirror-images are discerned by mental rotation. The M-system is the converse in both respects. A code of this system does not provide a handedness cue but, instead, a rotation cue. Thus, for handedness discrimination, the H-system neither needs nor guides mental rotation, whereas the M-system does both. This M-system generates object-centred structural codes enriched with viewpoint information. Various visual experiments reported in the literature favour the M-system over the H-system, implying that perception does not make use of an asymmetric but of a symmetric reference frame.

1 Introduction

In this paper we discuss the difference between images and mirror-images. Images are not taken here as the products of imagery, but as visual stimuli. These stimuli are either 2-D patterns or 3-D objects. In figure 1 some illustrations are given.

Figure 1B is the mirror-image of figure 1A, and vice versa. In their 2-D space, these patterns are, like the letters **d** and **b**, different. This means that, in their picture plane, these patterns cannot be mapped onto each other by translation or rotation transformations. This so-called handedness difference occurs only if each of the patterns is asymmetric. If this is not the case, ie if a 2-D pattern is bilaterally symmetric around whatever axis, it is identical to its mirror-image within the 2-D space. For instance, the horizontally symmetric letter **E** is identical to its mirror-image **Ǝ** even in 2-D space. The asymmetric figures 1A and 1B are still identical to each other if mapping is allowed through 3-D space: figure 1B, seen from the back, becomes figure 1A. This is not true of the images and mirror-images in figures 1C and 1D if they are regarded as objects. These objects are different and their handedness also differs in 3-D space. As a 2-D pattern in 2-D space, a 3-D object that is bilaterally symmetric around whatever axis, is equal to its mirror-image again in 3-D space. For instance, the objects in figures 1C and 1D, without chimneys, are equal to each other.

In this study we focus on the handedness of objects. The objects under investigation are corkscrews as presented in figures 2A and 2B. Figure 2A depicts a common corkscrew, to be turned clockwise for opening bottles. Thus, figure 2B can be called an anticlockwise corkscrew. All crucial characteristics of these corkscrews are represented, in a more stylised fashion, by the screws shown in figures 2C and 2D. In fact, we will only consider these screws because the handedness of any object can be represented by them. For instance, the three bottom contour segments of figure 1C agree with figure 2C, and the three bottom contour segments of figure 1D agree with figure 2D.

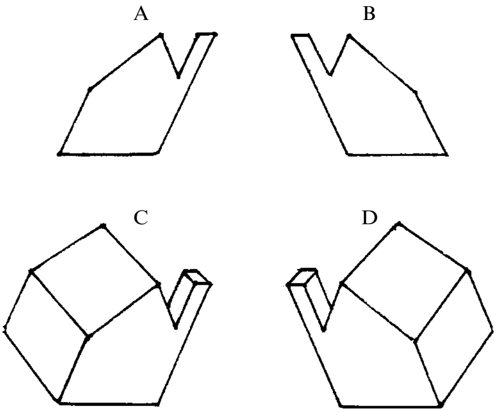

Figure 1. A is a 2-D image and B is a 2-D mirror-image. Within the 2-D picture plane these patterns are different. In 3-D space these patterns are equal: A, seen from the back, is B. C refers to a 3-D image and D to a 3-D mirror-image. In 3-D space these objects are different.

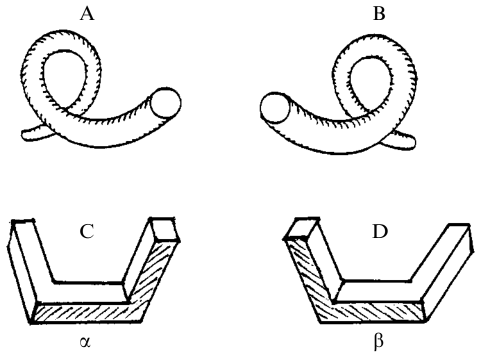

Figure 2. A is a common corkscrew and B is an uncommon corkscrew. C and D are stylised versions of, respectively, A and B. C is called an α screw and D is called a β screw.

Thus, the handedness of figures 1C and 1D is anchored by these screws. Elsewhere we have shown that these screws, which we call α and β screws, are the crucial components in the representation of 3-D solid shapes (Leeuwenberg and van der Helm 1991; Leeuwenberg et al 1994). This is why screws have often been the topic of vision research (Shepard and Metzler 1971; Pani et al 1996).

In vision research visual illusions have frequently been studied to discover general properties of perception and not because of the important role these illusions play in everyday perception. Many illusions cannot even be replicated outside the laboratory. The same is probably true for image – mirror-image discrimination. There are a few differently handed letters, namely **d** or **p** versus **b** or **q**, which should not be confused but, in everyday life, most differences between objects have nothing to do with handedness. Nevertheless, there is ample vision research on image – mirror-image discrimination. These studies do not so much focus on the direct visual relevance of that topic as on general properties of perception. The same holds for our approach to this topic. For instance, our study will clarify how codes are object-centred and structural, although they are still viewpoint-dependent (see later on).

The motive of this study has to do with a controversy of viewpoints on image – mirror-image discrimination in perception research. Our goal is to establish the most plausible viewpoint. As about the same controversy has taken place within philosophy but has lead to a clearcut conclusion, we will start with a brief review of this philosophical debate.

1.1 *Controversy among philosophers*

One of the first ideas on images and mirror-images stems from Kant (1783). He draws attention to the following paradox. All distances within an image are equal to those within its mirror image. For instance, the distances of 3, 4, and 5 cm between the corners of a triangle also hold for the mirrored triangle. This also applies to the angles of the triangle (Deutsch 1955). Yet, an image and a mirror-image may clearly be different. For Kant (1783) this difference is essential. Newton (1687) agrees with his view: space is not determined by spatial relations within an object, but by an absolute coordinate system beyond the objects. Accordingly, the space within a left-hand glove does not match with the space within a right-hand glove.

Leibniz (1714) has an opposite opinion. He considers the structure the images and mirror-images have in common to be essential and the difference accidental. This difference is merely a matter of viewpoint. The mirror-image of a 2-D pattern is equal to that pattern, seen from the back. This back side is only accessible by inversion in 3-D space. Analogously, the mirror-image of a 3-D object, for instance a left-hand glove, equals the opposite side of that glove, accessible by inversion in an hypothetical 4-D space. In fact, for a left-hand glove, the opposite configuration is the inside out of the right-hand glove. Freudenthal's (1962) conclusion is in line with Leibniz's view about this topic. The difference between left and right turns, which is crucial for the distinction between an image and a mirror-image, is not definable. He formally proved that one living being cannot explain the meaning of a left or right turn to another being without both having access to a common asymmetric structure. Hypothetically, if beings from another planet were completely equal to humans including their surroundings and assuming that we could communicate with them through a common language, then all distinctions could be made clear to them except the distinction between left and right turns. The reason is that the other planet might be the mirror-image of our planet. Thus, an asymmetric structure contains information about handedness that is not communicable, whereas a symmetric structure is fully communicable but contains no information about handedness.

In short, there are two options. According to Kant (1783) and Newton (1687) the difference between an image and a mirror-image is fundamental. According to Leibniz (1714) and Freudenthal (1962) this difference is accidental and basically indefinable. Instead, bilateral symmetries are definable and communicable. In fact, the view of Leibniz and Freudenthal is more commonly accepted than Kant's and Newton's notion (Weyl 1952; Gardner 1964).

1.2 *Controversy among perception researchers*

Here we will briefly outline our view on the dilemma among perception researchers. It will become clear that their dilemma agrees with the dilemma among philosophers. The whole discussion started with a study on image – mirror-image discrimination by Shepard and Metzler (1971). They showed that the reaction time needed to judge whether two shapes are identical or mirror-reversed versions increases linearly with their angular disparity, ie with the size of the smallest angle between the orientations of the two shapes. The experimental shapes are either 2-D patterns, or pictures of 3-D objects. Their conclusion is that handedness discrimination gives rise to an imagined rotation, or mental rotation of a pattern representation or pattern 'code'. Furthermore, two conclusions can be drawn about pattern codes. First, codes do not explicitly represent the handedness of patterns, otherwise mental rotation is superfluous. Second, if mental rotation applies to pattern codes, the rotation of these codes has to lead to the correct codes of the rotated patterns. Both conclusions are compatible with the assumption of Cooper (1976) and Finke (1980) that pattern codes involved in mental rotation are stimulus-analogous. A problem of such codes is that they miss cues indicating which rotation has to be tested in the process of matching patterns by mental rotation.

A different conclusion is drawn by Corballis (1988). On the basis of his experiments he assumes that, in an image – mirror-image discrimination task, resort is taken to 'mental rotation' indeed, but that visual codes nevertheless contain handedness cues. These cues are due to an innate asymmetric reference frame, such as a spiral or a rotating motion. The interaction between the frame and a pattern is assumed to establish the handedness of the pattern irrespective of its orientation. Thus, according to Corballis, visual codes are not stimulus-analogous, but describe classes of patterns both with respect to their handedness and their internal structure. In one or the other respect the conclusions of Corballis agree with those of Neisser (1967), Pylyshyn (1973), and Cohen and Kubovy (1993).

The parallel between the controversy in philosophy and in perception research is the following. According to Kant (1783) and Newton (1687) the difference between an image and its mirror-image is essential, and according to Corballis (1988) this difference is explicitly identified by a code. Thus, the view of Kant and Newton agrees with that of Corballis. According to Leibniz (1714) and Freudenthal (1962) there is no criterion for the difference between an image and its mirror-image, and according to Shepard and Metzler (1971) a visual code does not explicitly reveal this difference. Therefore, image – mirror-image discrimination needs mental rotation. Thus, the view of Leibniz and Freudenthal agrees with that of Shepard and Metzler.

As said, the view of Leibniz and Freudenthal is commonly accepted. However, it is still wrong to draw from there conclusions about perception, namely that the camp of Corballis is incorrect and that of Shepard and Metzler is correct. There are objections against the camp of Corballis indeed. For instance, it insufficiently explains the need for mental rotation. However, there are also objections against the opposite camp. For instance, stimulus-analogous codes are implausible. It is even unclear whether either camp is acceptable for perception anyway.

In order to reach a clear position with respect to the visual coding of images versus mirror-images, we make a code-theoretical detour: we discern two coding systems and consider their properties. The systems merely reflect the two opposed philosophical viewpoints. One system explicitly represents clockwise and anticlockwise turns by code labels, and is set up to explicitly represent pattern handedness. It is called the H-system, referring to handedness, and is paired to the views of Kant and Newton. The other system does not explicitly represent these two opposed turns. Instead, it explicitly represents bilateral symmetries. It is called the M-system, referring to mirror, and is paired to the views of Leibniz and Freudenthal. In all other respects the systems are equal. It is not our goal to propose general code systems for the representation of all kinds of shapes. The primary goal is more specific, namely to establish the properties of the systems precisely because of their handedness sensitivity (H-system) or insensitivity (M-system).

The systems will first be described without justification of chosen rules. Later, after the description of the systems, arguments are presented for the chosen rules by considering alternative rules. At the end, in the discussion, the two systems are evaluated by comparing their properties with some relevant experimental results on visual handedness discrimination taken from literature. The description of each system starts with a discussion of 2-D patterns as an introduction to 3-D objects.

2 The H-system

2.1 *2-D patterns*

The H-system makes use of an asymmetric frame for coding pattern handedness. This frame is a specific circular motion, here denoted as 'turn'. As frame we choose a clockwise turn. On the left of figure 3 this dynamic frame is presented by two clockwise turns, each starting from the centre. This centre applies to the horizontal axis and divides this axis into a left and right side. Thus, the dynamic frame is actually fixated positionally by a viewpoint-dependent static coordinate system.

Figure 3 also shows how simple contour components of 2-D surface patterns are represented by using this frame. One curve component is a hill shape and another is a valley shape. The turns start from the centre of each pattern curve, and are compared with the frame. A code is the series of matches and mismatches between these turns and the frame. We indicate a match by the symbol **h**, and a mismatch by the symbol **t**. The exact curvatures of the hill and valley shapes are disregarded. The hill shape in figure 3A consists of an anticlockwise and a clockwise turn. Thus, its code is a mismatch

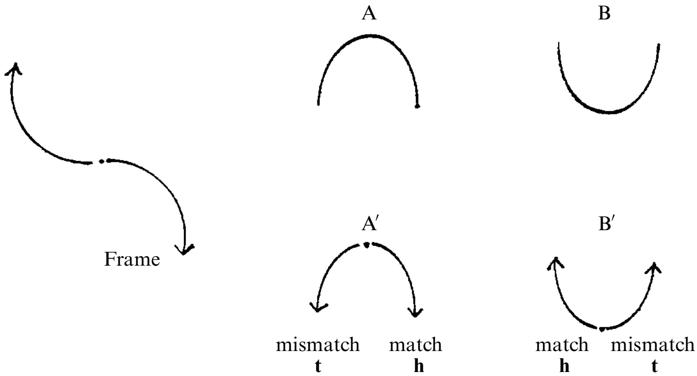

Figure 3. The frame of the H-system is depicted on the left: it is a clockwise turn. In fact, two versions of this turn are presented, each starting from the centre of the frame. A is a hill shape and B is a valley shape. The coding of these curves is illustrated in A′ and B′. The turns, starting at the centre of each curve, are compared with the corresponding turns of the frame. A′ delivers a mismatch and a match, and B′ delivers a match and a mismatch. A match is indicated by **h**, and a mismatch by **t**. Thus, the code of A = **th**, and the code of B = **ht**.

and a match, or **th**. The valley shape in figure 3B consists of a clockwise and an anticlockwise turn. Thus, its code is a match and a mismatch, or **ht**.

The given codes are serial. In analogy to the stimuli, they represent left-hand pattern parts on the left side and right-hand parts on the right side. Thus, the coding makes use of the coordinate system of the viewer, implying that the coding is viewpoint-dependent. This use is still at an implicit level, ie the code components do not explicitly describe left and right positions of pattern elements. They merely represent clockwise or anticlockwise turns on these positions (see later on) in a stimulus-analogous fashion. Thus, the difference between **th**, for the hill shape, and **ht**, for the valley shape, can only be established by mental rotation of these codes. However, the focus is how pattern differences can be established by code cues without mental rotation.

A property of the codes is concerned with their representation of symmetry and asymmetry. Here symmetry stands for bilateral symmetry around the vertical axis. An illustration is given in figure 3. The hill shape in figure 3A is symmetric but its code, **th**, is asymmetric. The combination of figures 3A and 3B, a meander pattern, is asymmetric, but its code, **thht**, is symmetric. In general, symmetric patterns are represented by asymmetric codes and asymmetric patterns by symmetric codes. We call this relation between patterns and their codes symmetry inversion. This symmetry inversion is related to the fact that the codes of the H-system provide a handedness cue. Before demonstrating this handedness cue by means of figure 4, we will first explain how the figure is composed.

Figure 4A is a standard pattern which is partly asymmetric. Figures 4B, 4C, and 4D are obtained by rotations of 180° of this standard pattern around the three axes z, x, and y, respectively. The same rotations apply to their codes. These axes are shown on the right in figure 4: z is the viewing axis and leads to a rotation in the picture plane; x is the medial axis; and y is the vertical axis. The latter two axes lead to rotations in 3-D space. Figure 4B is equal to figure 4A, whereas figures 4C and 4D are different from figure 4A in 2-D space. In this space, figures 4C and 4D are mirror-images of figure 4A. The codes describe only curves, ie they disregard line lengths.

2.2 *Handedness cue*

A handedness cue is a code property which characterises the handedness of a pattern. This property has to be assessable and has to be provided by each individual code. Here we assume that differently handed patterns are different in 2-D space. This means

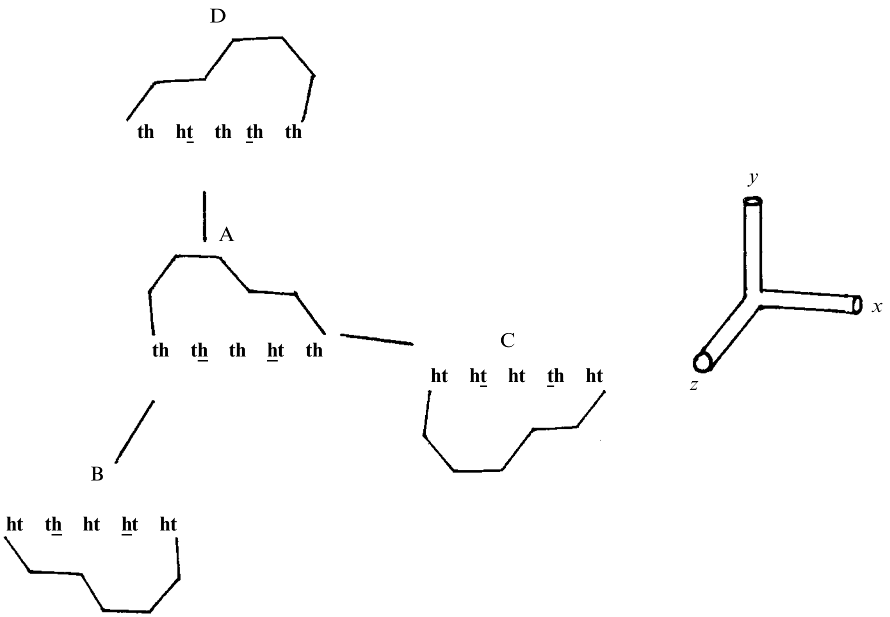

Figure 4. On the right, three axes are presented: z is the viewing axis, x is the medial axis, and y is the vertical axis. Three 180° rotations around these axes are applied to A. B is equal to A. C and D are equal to each other, but different from A of which they are mirror-images. The codes of the H-system contain a handedness cue. This cue is the first symbol which is equal to the other around the centre of each code. For A and B this symbol is **h**, for C and D this symbol is **t**.

that these patterns cannot be mapped onto each other by a translation and/or rotation in the picture plane. For the serial patterns in figure 4, the handedness is determined by the asymmetry around the centre of each pattern. This asymmetry is represented by symmetry of symbols in the codes. However, each pattern in figure 4 is only partly asymmetric. Thus, each code is only partly symmetric around its centre. The latter symmetry can easily be found by starting from the centre of the code (or from both ends) and then comparing the angular symbols that are symmetrically positioned around this centre. The first angular symbol that is equal to the other around this centre is a valid cue for the handedness of each pattern irrespective of its orientation. For instance, **h** is the cue for figure 4A as well as for figure 4B, whereas **t** is a cue for figure 4C as well as for figure 4D. These cues reveal that figure 4B is equal to figure 4A, and that figures 4C and 4D are equal to each other but, as mirror-images, different from figure 4A.

The codes of hill and valley shapes can be used in a code of the closed contour of a surface. This code is a cyclic series of symbols with the restriction that angles at the left side are represented on the left and angles at the right side are represented on the right in the code. There are two equivalent options, leading to codes which merely differ with respect to their 180° orientation in the picture plane. One is that the angles are represented as they appear from the outside of the closed contour. In that case a convex angle in the closed contour agrees with a valley shape and a concave angle with a hill shape. The other option is that the angles are represented as they appear from the centre of the closed contour. In that case a convex angle agrees with a hill shape and a concave angle with a valley shape. We choose the latter option. Furthermore, it is assumed that both sides of an angle have identical line segments, and that a curve that is neither convex nor concave but neutral in this respect is represented by the symbol **o**. In figure 5 contour codes are illustrated.

th ht th th ht ht th th o.o th ht th th ht th ht th **th ht th ht th th ht th o.o th th ht ht th th ht th**

Figure 5. The linear cyclic codes of the H-system unambiguously represent the surface patterns A and B. The code describes the contour as viewed from the inside of the pattern. In order to establish a handedness cue a unique anchor has to be specified. An appropriate anchor is the longest line segment. The first symbol which is equal to the other around the anchor provides a handedness cue. For A, this symbol is **h,** for B this symbol is **t**.

As the code of a closed contour is cyclic, any place on the contour can be chosen as the starting point or the end point of a code. Thus, the code does not provide a centre point between a starting and an end point that can function as an anchor for a handedness cue. Alternatively, for figure 5A, an appropriate anchor point is the neutral angle **o**. This neutral angle is unique both for figure 5A and for its mirror-image 5B. For figure 5A, the first symbol that is equal to the other around this neutral angle, in this case **h**, characterises the handedness of this figure. For figure 5B, the symbol **t** characterises the handedness of this mirror-image. Instances of other anchors are longest or shortest symmetries, longest or shortest asymmetries, or unique concavities. All such anchors are identifiable by pattern codes. Thus, pattern codes that have such anchors provide handedness cues.

2.3 *3-D objects*
As mentioned in the introduction, in this study we focus on basic elements of objects. The elements under investigation are the stylised α and β screws, shown in figures 2C and 2D. The coding recipe for such a screw is as follows. Each screw has three components and its code has therefore three symbols. The central component is indicated by **x**, **y**, or **z**, depending on whether the orientation of this central component agrees, respectively, with the x, y, or z axis. Thus, **x**, **y**, or **z** refer to screw axes. The two outer components of the screw are represented depending on how they are related to the central screw axes. One outer component refers to a 2-D aspect of a screw. It deals with the vertical component. If this vertical component, starting from the central screw axis, makes a clockwise turn, it is represented by **h**, and if this vertical component makes an anticlockwise turn, it is represented by **t**. The other outer component describes a 3-D aspect of a screw. It indicates whether an outer component of the screw is oriented forward or backward. Forward is indicated by **f**, and backward by **b**.

For a screw image, presented in figure 6A, the given coding is rather straightforward. As its central component agrees with the x axis, it is clear how the 2-D and 3-D turns of the outer components should be interpreted, ie clockwise or anticlockwise and forward or backward. However, for screw images, shown in figures 6B and 6C, whose central components agree with the y and z axis, respectively, the coding is less obvious. Therefore we assume an intermediate step in which these images are turned toward a canonical orientation (Pani et al 1996). This orientation is such that screw axes are aligned with the x axis. Instead of turning images, an equivalent step for obtaining this canonical alignment is to turn viewers as shown in figure 6. Their orientation with respect to screw axes has to be the same as the orientation of the viewer of figure 6A with respect to its axis. As for this x axis screw the viewer is aligned with the y axis, for the y axis screw the viewer should be aligned with the z axis, and for the z axis screw the viewer should be aligned with the x axis. As a consequence, the configuration of viewers and axes is rotation invariant.

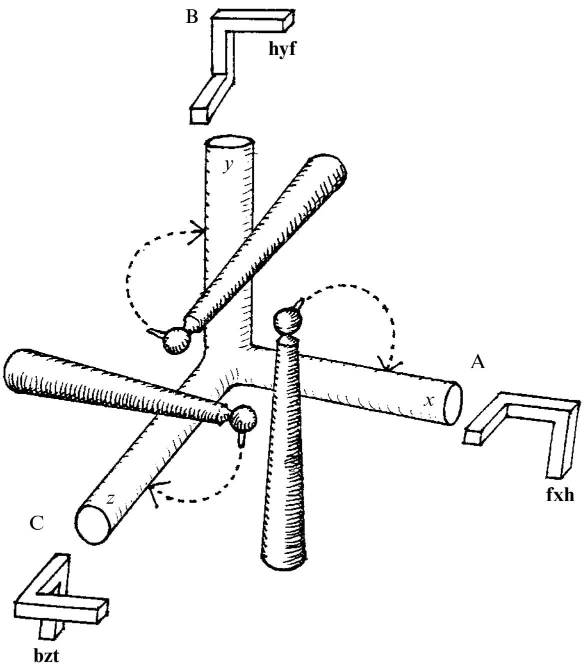

Figure 6. An illustration of H-system coding. A, B, and C are screw images with varying central axes. These axes are represented by *x*, *y*, and *z* in the centres of their codes. For representing the outer components of the screws, different viewers are assumed for each screw axis. A *y*-oriented viewer for an *x* screw axis, an *x*-oriented viewer for a *z* screw axis, and a *z*-oriented viewer for a *y* screw axis. **h** refers to a clockwise turn, **t** to an anticlockwise turn, **f** to a forward screw component, and the **b** to a backward screw component. The dotted turns, being clockwise, refer to backward rotations.

In the discussion we will argue that alternative viewer–axis configurations are possible which are equivalent with respect to the properties we consider here. All acceptable configurations specify the backward rotation of a screw. Note that this backward rotation is unrelated to clockwise or anticlockwise rotation, and is unrelated to the asymmetric frame. The backward rotation, indicated in figure 6 by dotted turns, accidentally appears to be clockwise merely because the screws are depicted as seen from the right side. In figure 7 these backward rotations are applied to an α screw and a β screw, in 90° steps, around all *x*, *y*, and *z* axes. The starting image of the α screw is figure 7A, and the starting image of the β screw is figure 7K.

2.4 *Handedness cue*

All codes of figures 7A to 7J, ie of the α screw, are characterised by specific combinations of symbols referring to the outer screw components: either **fh** or **bt**. The screw-axis components are irrelevant. Similarly, all codes of the mirror figures 7K to 7T, ie of the β screw, only comprise **ft** or **bh** combinations. These combinations are identifiable and therefore valid cues for distinguishing between α and β screws independent of viewers. This means that all single screws in figure 7 can be identified as α or β screws on the basis of codes without mental rotation.

The handedness cues for single screws allow us to establish the handedness of a chain of elementary screws. Such a chain is shown in figure 8A. It consists of three overlapping screws—figures 8B, 8C, and 8D—which can be identified as an ααβ series. The next step is analogous to the procedure we have proposed for 2-D patterns. It entails starting from the centre of the series (or from both ends) and subsequently testing the identified screws which are symmetrically positioned around this centre.

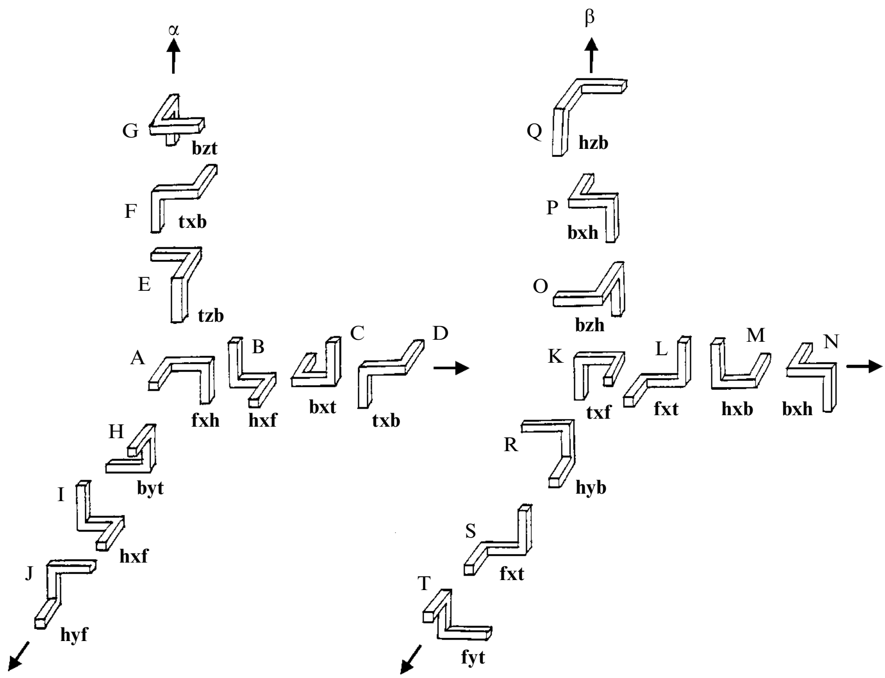

Figure 7. From A to J, all 90° backward rotations are presented of an α screw around the x, y, and z axes. From K to T, all 90° backward rotations are presented of a β screw around the x, y, and z axes. The codes of A to J are characterised by **f h** or **b t** combinations of symbols, and the codes of K to T are characterised by **f t** or **b h** combinations of symbols. These combinations are identifiable and therefore valid cues to distinguish between α and β screws. However, the codes are not sufficiently differentiated for providing a rotation index.

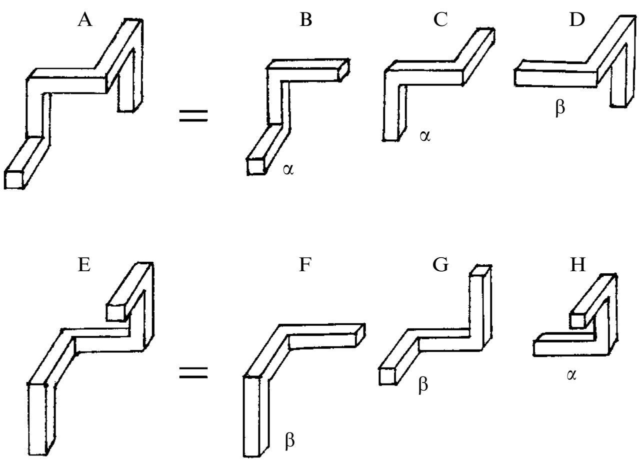

Figure 8. A is a composition of B, C, and D screws: A = α α β. E is a composition of F, G, and H screws: E = β β α.

The handedness cue of the chain is the kind of screw in the centre, or the kind of screw that is symmetrically positioned around this centre. This cue is valid irrespective of the orientation of the object. For instance, figure 8A is characterised by the centre screw α, whereas the mirror-image of figure 8A, namely figure 8E, is characterised by the centre screw β. This implies that mental rotation is not required to assess the handedness of such complex objects.

2.5 *Insufficient rotation index*

For mental rotation a rotation index is useful. This index is a code cue which indicates beforehand which rotation has to be verified by code rotation to establish whether two shapes are equal. The codes of the H-system provide some information about the axis of rotation. If two screw images have the same central axis, the rotation is around this axis. For instance, figures 7B and 7C are, respectively, represented by the codes **hxf** and **bxt**. These codes share the *x* axis. Hence, the two images are related by a rotation around the *x* axis. Furthermore, if two screw images have different screw axes, the rotation is around the third axis. For instance, figures 7A and 7E are, respectively, represented by the codes: **fxh** and **tzb**. Hence, the two images are related by a rotation around the *y* axis.

The codes of the H-system do not tell us yet whether a rotation is backward or forward. First of all, there are various codes for different screw images which consist of the same combinations of symbols. Note that the system is insensitive to the order of symbols. For instance, the codes of figures 7A and 7B are equal. The same applies to the codes of figures 7C and 7D, and of figures 7E and 7G. Besides, for pairs of different codes there is no consistent rotation index. For instance, the transition from code **byt** to code **hxf** corresponds to a backward rotation of figure 7H to 7I, but the 'same' code transition, namely from code **byt** to code **fxh** corresponds to a forward rotation of figure 7H to 7A. This kind of inconsistency applies to all pairs of different codes. In sum, the codes insufficiently provide rotation cues.

3 The M-system

3.1 *2-D patterns*

The basic assumption is that the M-system is insensitive to the difference between clockwise and anticlockwise turns. Thus, its codes lack labels for these asymmetric concepts. Instead, its codes are sensitive to bilateral symmetry. In all other respects, the M-system is similar to the H-system. For instance, the M-system also makes use of a reference frame. This frame, depicted on the left in figure 9, consists of a symmetric pair of opposed circular motions, here denoted as 'turns'. As a frame we chose an anticlockwise and a clockwise turn, each starting from the centre. This centre applies to the horizontal axis and divides this axis into a left and a right side. Thus, the dynamic frame is actually fixated positionally by a viewpoint-dependent static coordinate system.

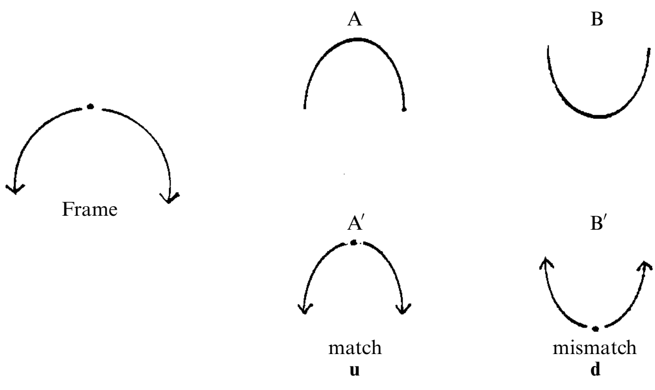

Figure 9. The frame of the M-system is depicted on the left: it is an anticlockwise and a clockwise turn, each starting from the centre of the frame. A is a hill shape and B is a valley shape. The coding of these curves is illustrated in A′ and B′. The turns, starting at the centre of each curve, are compared with the corresponding turns of the frame. A′ delivers a match, and B′ delivers a mismatch. A match is indicated by **u**, and a mismatch by **d**. Thus, the code of A = **u**, and the code of B = **d**.

Figure 9 also shows how a hill shape and a valley shape are represented by using this frame. The turns, starting from the centre of each pattern curve, are compared with the pairs of turns of the frame. A code is the series of matches and mismatches between the turns of the pattern and the turns of the frame. The exact magnitude of their curvatures is disregarded. We indicate a match by the symbol **u**, and a mismatch by the symbol **d**. The hill shape, in figure 9A, consists of an anticlockwise and a clockwise turn. Thus, its code is a dual positive match. This is simply indicated by a single **u**. The valley shape, in figure 9B, consists of a clockwise and an anticlockwise turn. Thus, its code is a dual mismatch. This is simply indicated by a single **d**.

Like the codes of the H-system, the codes of the M-system are serial. In analogy to the stimuli, they represent left-hand pattern parts on the left side and right-hand parts on the right side. Thus, the coding makes use of the co-ordinate system of the viewer, implying that the coding is viewpoint-dependent. This use is still at an implicit level, ie the code components do not explicitly describe left and right positions of pattern elements. They merely represent clockwise or anticlockwise turns on these positions (see later on) in a stimulus-analogous fashion.

A property of the codes concerns their representation of symmetry and asymmetry. Here symmetry stands for bilateral symmetry around the vertical axis. An illustration is given in figure 9. The hill shape in figure 9A is symmetric, as is its code **u**. The combination of figures 9A and 9B, a meander pattern, is asymmetric, as is its code **ud**. In general, symmetric patterns are represented by symmetric codes and asymmetric patterns by asymmetric codes. Thus, the codes do not reveal symmetry inversion. Owing to this property, the codes do not provide a handedness cue. We will demonstrate this by means of figure 10, which is identical to figure 4 except for the codes.

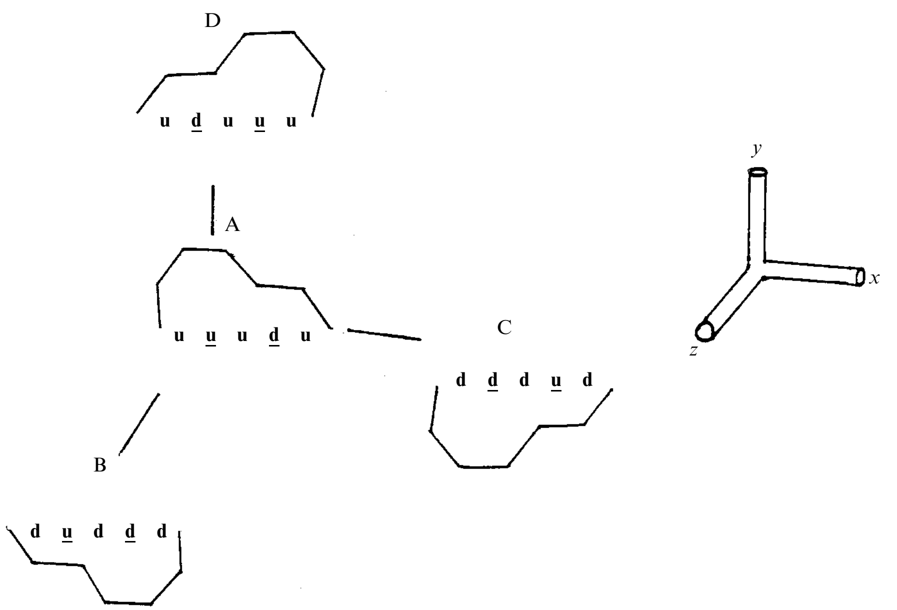

Figure 10. Three 180° rotations around the z, x, and y axes (shown on the right) are applied to A. B is equal to A. C and D are equal to each other, but different from A of which they are mirror-images. For the M-system, the first two different symbols around the centre of each code characterise the handedness of a pattern. For A and B these symbols are **ud**, for C and D these symbols are **du**. However, permutations of symbols are not assessable from each code. Hence, codes of the M-system do not provide handedness cues.

3.2 *No handedness cue*

A handedness cue is a code property which characterises the handedness of a pattern. This property has to be provided by each individual code, and has to be assessable by the M-system. We suppose that differently handed patterns are different in 2-D space, ie these patterns cannot be mapped onto each other by a translation and/or rotation in the picture plane. For the serial patterns in figure 10, this handedness is determined by asymmetry around the centre of a pattern. This asymmetry is represented by asymmetry in the code. However, each pattern in figure 10 is only partly asymmetric. Thus, each code is only partly asymmetric around its centre. To find the latter asymmetry, one can start from the centre of the code (or from both ends) and then test the symbols being symmetrically positioned around this centre. The goal is to establish the pair of angular symbols which are unequal. This pair of symbols is characteristic for the handedness of each pattern irrespective of its orientation. For instance, **u d** applies to figure 10A as well as to figure 10B, whereas **d u** applies to figure 10C as well as to figure 10D. The two pairs contain the same symbols in different serial orders.

The question is whether these pairs of symbols are valid handedness cues. If so, they should be provided by each individual code and be assessable. The identification of a **u d** pair of symbols in one code, as distinct from a **d u** pair in another code, requires sensitivity to reading order. However, the codes of the M-system do not explicitly represent left versus right positions of pattern elements. In other words, they are insensitive to reading order, as they are insensitive to asymmetry anyway. The consequence is that, for image and mirror-image discrimination, the code of one pattern has to be rotated towards the code of the other pattern. If both codes can be matched, the patterns are equal; otherwise they are unequal. Thus, mental rotation is needed for handedness identification. This applies to serial patterns but also to surfaces as shown in figure 11. Like the codes of the H-system in figure 5, the codes of the M-system represent closed contours by linear series of symbols conceived as cyclic strings. The only difference is that convex angles are represented by **u** and concave angles by **d**.

u d u u d d $\underline{\text{u}}$ u o.o u $\underline{\text{d}}$ u u d u d u **u d u d u u $\underline{\text{d}}$ u o.o u $\underline{\text{u}}$ d d u u d u**

Figure 11. The linear cyclic codes of the M-system unambiguously represent the surface patterns A and B. The code describes the contour as viewed from the inside of the pattern. In order to establish a handedness cue, a unique anchor has to be specified. An appropriate anchor for A and its mirror-image B is the longest line segment. The first pair of different symbols around the centre of this anchor characterises the handedness of a pattern. For A these symbols are **u d**, for B these symbols are **d u**. However, these permutations of symbols are not valid handedness cues.

3.3 *3-D objects*

The M-system represents screw images, shown in figures 2C and 2D, in about the same way as the H-system. The coding recipe is as follows. Each screw has three components and its code has therefore three symbols. The central component is indicated by **x**, **y**, or **z**, depending on whether the orientation of this component agrees with respectively the x, y, or z axis. The two outer components of the screw are represented depending on how they are related to the central screw axes. One outer component refers to a 2-D aspect of a screw. It deals with the vertical component. If this vertical component, starting from the central component, makes an anticlockwise turn on the left side or a clockwise turn on the right side, the turn matches with the frame and is therefore represented by **u**. In this case the combination of the vertical and the central components constitutes a hill curve. Equally, if the vertical component, starting from the central component, makes a clockwise turn on the left side or an anticlockwise turn on the right side, the turn does not match with the frame and is therefore represented by **d**. In that case the combination of the vertical and the central components constitutes a valley curve. The other outer component describes a 3-D aspect of a screw. It indicates whether an outer component of the screw is oriented forward or backward. Forward is indicated by **f**, and backward by **b**.

For a screw image presented in figure 12A the given coding is rather straightforward. As its central component agrees with the x axis, it is clear how the 2-D and 3-D turns of the outer components should be interpreted, ie clockwise or anticlockwise and forward or backward. However, for screw images shown in figures 12B and 12C, whose central components agree with the y axis and the z axis, respectively, the coding is less obvious. Therefore, we assume an intermediate step in which these images are turned toward a canonical orientation (Pani et al 1996). This orientation is such that screw axes are aligned with the x axis. Instead of turning images, an equivalent step for obtaining this canonical alignment is to turn viewers as shown in figure 12. Their orientation with respect to screw axes has to be the same as the orientation of the viewer of figure 12A with respect to its axis. As for this x axis screw the viewer is aligned with the y axis, for the y axis screw the viewer should be aligned with the z axis, and for the z axis screw the viewer should be aligned with the x axis. As a consequence, the configuration of viewers and axes is rotation-invariant.

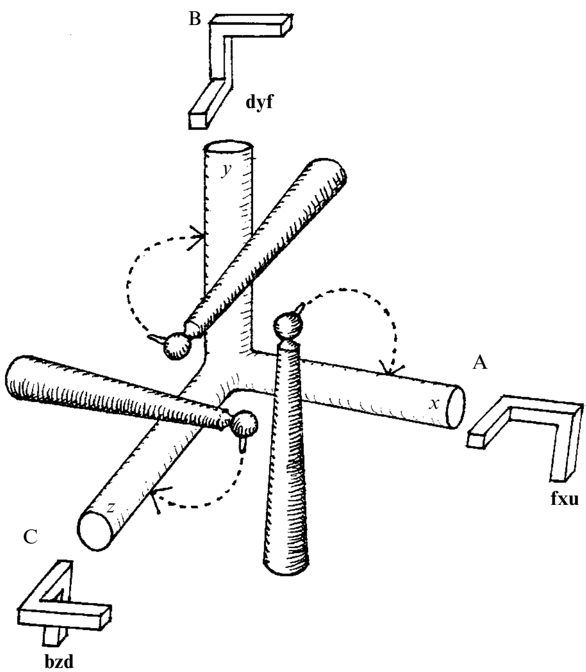

Figure 12. An illustration of M-system coding. A, B, and C are screw images with varying central axes. These axes are represented by x, y, and z in the centres of their codes. For representing the outer components of the screws, different viewers are assumed for each screw axis. A y-oriented viewer for a x screw axis, an x-oriented viewer for a z screw axis, and a z-oriented viewer for a y screw axis. **u** refers to an anticlockwise turn at the left side or a clockwise turn at the right side; **d** refers to a clockwise turn at the left side or an anticlockwise turn at the right side; **f** refers to a forward screw component, and **b** to a backward screw component. The dotted turns, being clockwise, refer to backward rotations.

In the discussion we will argue that alternative viewer–axis configurations are possible which are equivalent with respect to the properties we consider here. All acceptable configurations specify the backward rotation of a screw. Note that this backward rotation is unrelated to clockwise or anticlockwise rotation. The backward rotation, in figure 12 indicated by dotted turns, accidentally appears to be clockwise merely because the screws are depicted as seen from the right. In figure 13 these backward rotations are applied to an α screw and a β screw, in 90° steps, around all x, y, and z axes. The starting image of the α screw is figure 13A, and the starting image of the β screw is figure 13K.

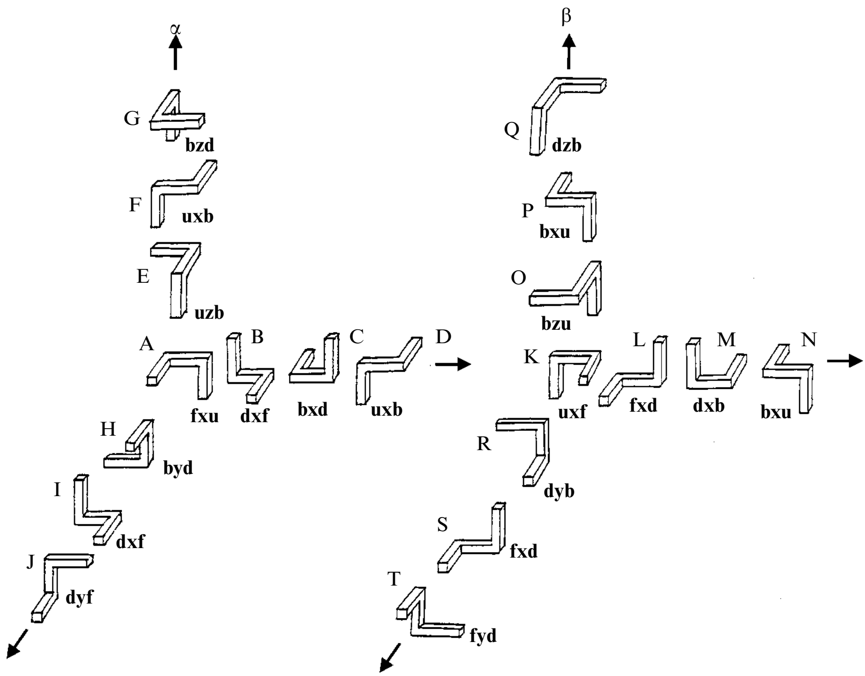

Figure 13. From A to J, all 90° backward rotations are presented of an α screw around the x, y, and z axes. From K to T, all 90° backward rotations are presented of a β screw around the x, y, and z axes. The codes of A to J are not characterised by unique combinations of symbols as distinct from those in the codes of K to T. Therefore, the codes do not provide distinctive cues of α and β screws. Instead, the codes are sufficiently differentiated to provide a rotation index.

3.4 *No handedness cue for 3-D objects*

The codes of figures 13A to 13J, ie of the α screw, consist of all possible combinations of code symbols: **f** is combined both with **u** and **d**, and β is combined both with **u** and **d**. The same applies to the codes of mirror-figures 13K to 13T, ie of the β screw. As a consequence, the difference between the α and β codes is merely determined by the difference between permutations of code symbols. This holds independently of viewers. Thus, the codes of each kind of screw are not characterised by unique and identifiable combinations of symbols. Therefore mental rotation is needed for image–mirror-image discrimination.

3.5 *Rotation index*

Another issue is whether codes provide a rotation index, indicating beforehand the kind of rotation which has to be verified to establish whether two shapes are equal. The codes of the M-system provide this index. First, like the codes of the H-system, the codes of the M-system supply information about the axis of rotation. If two screw images have the same central axis, the rotation is around this axis. For instance, figures 13B and 13C are, respectively, represented by the codes **dxf** and **bxd**. These codes share the x axis. Hence, the two images are related by a rotation around the x axis. Furthermore, if two screw images have different screw axes, the rotation is around the third axis. For instance, figures 13A and 13E are, respectively, represented by the codes: **fxu** and **uzb**. Hence, the two images are related by a rotation around the y axis.

Unlike the codes of the H-system, there are no same codes for different screw images. Besides, the codes also provide consistent cues, being the same for α and β screws, indicating whether a rotation is 90° backward or forward. We discern two kinds of cues for two different cases. In one case two screw images have the same central axis, and in the other case they have different central axes. Each case will be considered separately.

In the case when two images have the same screw axis, and their codes share the symbol **d**, the code with the symbol **b** refers to the 90° backward-rotated screw image. We indicate this sequence by: 2**d** → 1**b**. For instance, the figures 13B and 13C have the same central axis, namely the x axis. Their codes are respectively **dxf** and **bxd**. These codes share the symbol **d**. The second code has the symbol **b**. Hence, the latter code refers to the 90° backward (clockwise) rotated screw image, namely figure 13C. The same property holds for 2**b** → 1**u**, 2**u** → 1**f**, and 2**f** → 1**d**. In figure 14, this cue is more simply indicated in a cyclic fashion.

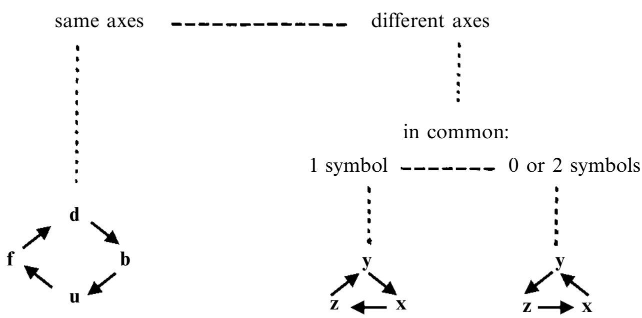

Figure 14. A schematic presentation of the code cue for backward rotation. This cue is provided by the M-system.

In the case when two screw images have different central axes, there are two options. Either their codes share precisely one symbol or do not share precisely one symbol. We will elaborate on these two options next.

If two codes, with different axes, share precisely one symbol, the 90° backward rotation direction is specified by the sequence: **y** → **x** → **z** → **y**. In figure 14, this sequence is indicated in a cyclic fashion. For instance, figures 13A and 13E have different central axes, namely x and z. Their codes are, respectively, **fxu** and **uzb**. These codes share just one symbol, namely **u**. The second code has the symbol **z**. Thus, the latter code refers to the 90° backward-rotated screw image, namely figure 13E.

If two codes, with different axes, do not share just one symbol, but zero or two symbols, the 90° backward-rotation direction is specified by the sequence: **y** ← **x** ← **z** ← **y**. In figure 14, this sequence is indicated in a cyclic fashion. For instance, figures 13E and 13F have different central axes, namely z and x. Their codes are, respectively, **uzb** and **uxb**. These codes share two symbols. The second code has the symbol **x**. Thus, the latter code refers to the 90° backward rotated screw image, namely figure 13F.

The given rotation cue is rather complex indeed. Therefore we assume that, if this cue is actually used in handedness discrimination tasks, some learning might be needed. Nevertheless, the rotation cue is not only consistent for both α and β screws, but also for 90° rotations of starting screw images other than the one in figure 13. In figure 13, the 90° rotations of the α screw are presented for a starting screw image, figure 13A, whose central component coincides with the x axis. In figure 15, these 90° rotations of the α screw are shown for a starting screw image, figure 15A, whose central component coincides with the y axis, and one whose central component coincides with the z axis—figure 15K.

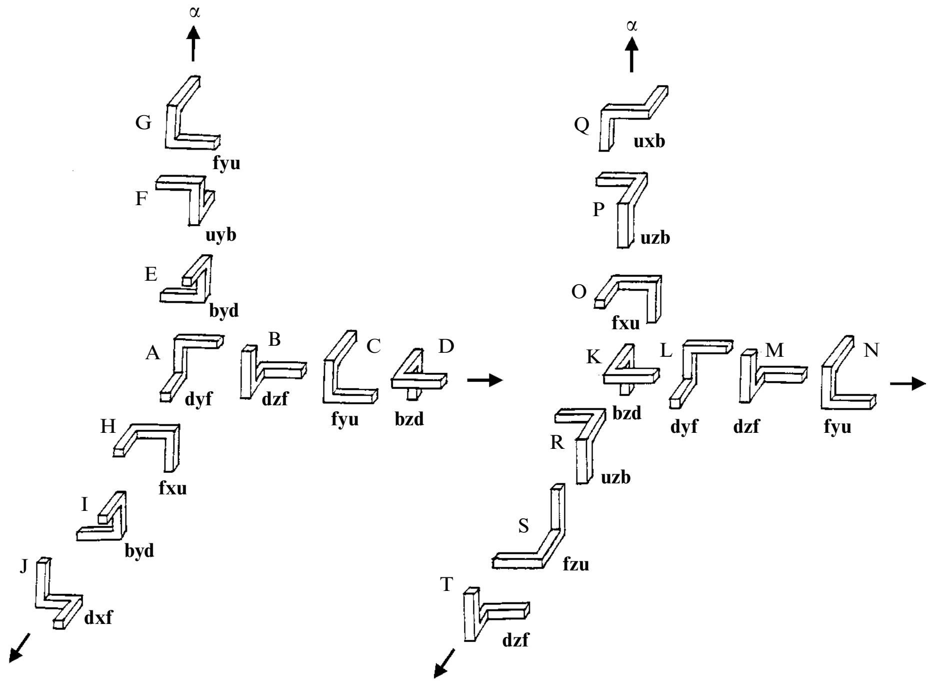

Figure 15. The central component of A agrees with the y axis, and the central component of K agrees with the z axis. From A and from K, all 90° backward rotations are presented of an α screw around the x, y, and z axes. Each subsequent pair of codes of the M-system provides a distinctive cue for backward versus forward rotation.

4 On assumptions of the systems

So far we have presented each system separately without justification of their assumptions. Here we will focus on the arguments and consider alternative assumptions. As the two systems differ merely with respect to their frames, the same justification largely applies to both systems.

4.1 *Viewpoint-dependent frames*

Not only the codes of the H-system but also the codes of the M-system are handedness-sensitive. The difference is that the codes of the H-system explicitly represent pattern handedness, whereas the codes of the M-system represent pattern handedness implicitly. The latter system reveals handedness by matching rotated codes. As argued in the introduction, handedness is a viewpoint-dependent property of patterns. Therefore, the turn frame of each system is fixated by a viewpoint-dependent static coordinate system.

This coordinate system divides the frame into a turn at the left side and a turn at the right side on the horizontal axis, and with respect to these two sides the matching takes place of the pattern turns. This may suggest that the resulting serial code describes left and right positions of pattern elements. But this is wrong. The code represents, in analogy to the stimulus, left-hand pattern parts on the left side and right-hand parts on the right side. In other words, the code does not explicitly represent left and right positions of pattern elements. It merely represents clockwise or anticlockwise turns on these positions.

4.2 *Turn frames*

A feature the two frames share is that they deal with specific circular motions, here indicated as 'turns'. We claim that, in order to represent the handedness of both 2-D and 3-D shapes, either explicitly or implicitly, 2-D asymmetric structures, such as turns, are essential. 1-D left and right directions alone are insufficient as components of frames for discerning images from mirror-images. The reason is that directions do not constitute unique and stable characteristics of images as distinct from mirror-images. For instance, the left part of a pattern is identical to the right part of the same pattern turned upside down in the picture plane. Furthermore, frames consisting of angles and/or distances do not even contain handedness information at all (Deutsch 1955).

To some extent, the choice of turn frames is free. For instance, we chose a clockwise turn frame for the H-system, but an anticlockwise turn frame would have served equally well. The range of equivalent, alternative frames is wider for the M-system. Here, we chose a divergent pair of an anticlockwise turn and a clockwise turn, but a divergent pair of a clockwise and an anticlockwise turn would have been just as suitable. Convergent pairs of opposed turns, starting from two different points, are also equally acceptable. What matters is that the frame of the H-system is asymmetric and the frame of the M-system is symmetric.

4.3 *Anchoring shapes to frames*

In general, symmetric variations of the systems do no harm. In contrast, adding asymmetry, of whatever kind, may change the system drastically. Illustrations will be given next.

To obtain the code of a 2-D pattern, the turns starting from the centre of a curve are compared with the turns of the frame. This is the method we opted for, but one could as easily take the turns that start from both ends of each curve and compare those with the ones of the frame. For a 3-D screw the same holds. For obtaining its code, the same turns, starting from the centre, are compared with the turns of the frame. This choice is also free, provided that the point at which scanning is started is not asymmetric.

An asymmetric start of scanning a curve, say from the left to the right, is not useful, because then both systems become indiscernible M-systems. For the M-system, the asymmetry of the start has two consequences which compensate each other. First, only the clockwise turn of its frame is then of use. The result is that the M-system changes into the H-system with an asymmetric start. Second, the asymmetry of this start compensates the asymmetry of the frame of the H-system. Thus, the effects of an asymmetric scanning are that the M-system remains unchanged and the H-system changes into the M-system.

4.4 *Viewers of screws*

In figures 6 and 12, a specific arrangement of viewers is chosen for screws with varying axes. All viewers are in front of the axes. Besides, all viewers agree with these axes: a y-oriented viewer for an x screw axis, an x-oriented viewer for a z screw axis, and a z-oriented viewer for a y screw axis. In this way, the viewer–axis arrangement is rotation-invariant, guaranteeing that the representation of a screw is independent of its screw axis. Nevertheless, there are other arrangements which are equally correct. A candidate is: the x-oriented viewer for the y axis, the y-oriented viewer for the z axis, and the z-oriented viewer for the x axis. Furthermore, viewers are admitted whose head and tail are reversed and which are behind the axes. This implies that eight different rotation-invariant arrangements are equally valid.

4.5 *Correct coding under rotation*

The assumption can be made that not only a code of the M-system but also a code of the H-system has to be appropriate for mental rotation. In that case, it holds for both systems that a literal rotation of the code of a pattern leads to an adequate code of an equally rotated pattern (see introduction). This property applies if rotation (R) and coding (C) are commutative, or formally: $\mathrm{R\,C} = \mathrm{C\,R}$. For the two systems the consequences are different. We will illustrate this property for each system under the hypothetical condition that codes consist of symbols and that these symbols are enriched with a specific semantic connotation.

The H-system discerns a clockwise turn, indicated by **h**, and an anticlockwise turn, indicated by **t**. Suppose that **h** stands for the head and the **t** the tail side of a coin. A 180° rotation in the picture plane of a clockwise or an anticlockwise turn does not change these turns. The same applies to the head or tail of a coin. In contrast, the back side of a clockwise turn is an anticlockwise turn and vice versa. This reversal also applies to the head or tail of a coin under this condition.

The M-system discerns, though indirectly, a hill shape, indicated by **u**, and a valley shape, indicated by **d**. Suppose that **u** stands for an upward orientation and **d** for a downward orientation. A 180° rotation in the picture plane of a hill shape leads to a valley shape and vice versa. Equally, this reversal applies to upward or downward orientations under this condition. In contrast, the back side of a hill shape is still a hill shape, and the back side of a valley shape is still a valley shape. This is also true for upward or downward orientations under this condition.

In fact, we do not assume that real neural codes in the brain consist of symbols, let alone symbols with semantic connotations, but we do believe that neural codes indeed behave analogously to the properties of the proposed codes consisting of symbols with the given semantic connotations.

5 Discussion

In order to establish which system is visually most relevant, we will compare the properties of the systems with some related experimental results on visual-handedness discrimination taken from literature. Note that the properties of the systems do not have the same status. Those with respect to handedness actually determine the systems themselves. This means that the H-system is designed to have handedness-sensitive codes and the M-system to miss such codes. This is not true for the properties with respect to rotation indices. These are consequences of the systems. In other words, the systems are not set up to have or to miss these indices.

Before discussing the visual relevance, we will briefly make clear why the systems have opposed properties. With this aim let us consider relations between codes of the two systems. These codes represent primitive motions. The relations between these codes are expressed by the following equations:

$$\mathbf{h} = \mathbf{d\,u}, \quad \mathbf{t} = \mathbf{u\,d};$$

$$\mathbf{u} = \mathbf{t\,h}, \quad \mathbf{d} = \mathbf{h\,t}.$$

The two upper equations describe a clockwise and an anticlockwise turn. The H-system describes these turns by single terms, the M-system by pairs of terms. The two bottom equations represent a hill and a valley curve. The H-system describes these curves by pairs of terms, the M-system by single terms. Thus, both pairs of equations reveal that what is expressed by one term in one system is expressed by pairs of terms in the other system and vice versa. This implies that combinations (single symbols) in one system are permutations of two symbols in the other system and vice versa. In other words, cues (combinations) in one system are non-cues (permutations) in the other system and vice versa.

5.1 *Handedness cue*

The codes of the H-system provide handedness cues, whereas the codes of the M-system do not. Thus, for image – mirror-image discrimination, the H-system actually does not need code rotation, whereas the M-system does. The studies of Corballis (1988) and of Cohen and Kubovy (1993) favour the H-system. These workers argue that the handedness of some 2-D patterns can be established without mental rotation. Cohen and Kubovy (1993) show that image – mirror-image discrimination experiments do not provide sufficient evidence to establish mental rotation as the only instrument for handedness identification. Experiments by Corballis show that rotations of ordinary letters are more quickly identified than rotations of their mirror versions. In his view, this outcome supports that recognition of pattern handedness does occur independently of pattern orientation, and without mental rotation. In fact, Corballis assumes that this handedness is established at an implicit level, but that for the explicit discrimination of images from mirror-images resort is still taken to mental rotation.

To us it is not clear why resort should be taken to mental rotation if pattern codes are handedness-sensitive. In our view, it is more plausible that the high recognition rates are due to the fact that familiar letters are overlearned, rather than that they are the result of the handedness-sensitivity of their codes.

There are also experiments that support the M-system. One such study is by Shepard and Metzler (1971), the pioneers of research into image – mirror-image discrimination. As indicated earlier, they have shown that the reaction time needed to judge whether two shapes are identical or mirror-reversed versions increases linearly with their angular disparity, ie with the size of the smallest angle between the orientations of the two shapes. Their experimental shapes are either 2-D patterns, or pictures of 3-D objects. Their conclusion is that handedness discrimination gives rise to an imagined rotation, or mental rotation. If we assume that the literal rotation of pattern codes of the M-system needed for image – mirror-image discrimination corresponds with mental rotation, the M-system is in line with these results. The H-system is not supported, as its codes provide handedness cues. Indeed, the H-system provides these cues only for those 2-D surface patterns that contain both convex and concave angles, but also for these patterns mental rotation appears to be needed for image – mirror-image discrimination.

There is still a gap between the M-system and the view of the initial researchers on mental rotation. Shepard and Metzler (1971) assume that mental rotation applies to representations of imagined patterns. Here we call these representations imagery codes. Furthermore, Cooper (1976), Finke (1980), and many others who have done similar experiments, tentatively conclude that imagery codes involved in image and mirror-image tasks have a stimulus-analogous character. A completely stimulus-analogous pattern code represents a class containing just one element, namely the stimulus pattern itself, and not a class of many patterns. Put simply, such a stimulus-analogous code does not classify patterns. However, in contrast to an imagery code, a code of the M-system classifies patterns. This discrepancy gives rise to various questions. One question is whether imagery codes are related, or even equal, to codes of actually presented stimuli?

There is some indirect empirical evidence for a positive answer. The experiments of Moyer (1973), Petersen (1975), and Kosslyn (1975, 1981) show that effects of imagined patterns are similar to those of actual stimuli. In line with these findings, recent new evidence from studies involving patients with brain damage and from measurements of regional brain activity suggests that many modality-specific areas, especially in the occipital cortex, are not only used in visual perception but also in imagery (Tippett 1992;

Farah 1995). The conclusions of these studies may also apply to amodal completion. We think that amodal completion and imagery are similar. Kanizsa and Gerbino (1982) have shown that this completion may occur at a visual level as distinct from higher cognitive levels. Furthermore, various experiments of Gerbino and Salmaso (1987), Sekuler and Palmer (1992), and van Lier et al (1995) provide evidence for the functional equivalence between amodal completion and visual perception. This completion is very consistent among subjects and not arbitrary. Sekuler and Palmer (1992) show that this amodal completion is achieved within about 400 ms.

All these studies seem to corroborate the assumption that imagery codes are related or even equal to visual codes of actual patterns. If this is true and these visual codes do indeed classify patterns, it is plausible that imagery codes also classify patterns. In the literature on perception research there is ample evidence that shows that visual codes classify patterns (Neisser 1967; Garner 1974; Palmer 1977; Restle 1982; Rock 1983; Collard and Buffart 1983).

Yet, the question is whether codes involved in mental rotation are stimulus-analogous? A direct empirical answer has been provided by Pylyshyn (1973). He shows that mental rotation is affected both by pattern regularity and context. However, stimulus-analogous codes are insensitive to pattern regularity and context. Hence, it is not plausible that codes involved in mental rotation are stimulus-analogous. A theoretical argument is that stimulus-analogous codes do not provide a cue for the rotation to be tested by mental rotation in the case of mirror discrimination. However, the existence of such a cue is plausible, otherwise innumerable rotations would have to be tested by mental rotation, especially in case of 3-D shapes. Therefore, it is plausible that codes involved in mental rotation are not stimulus-analogous.

The latter conclusion is consistent with the M-system: it needs mental rotation for image – mirror-image discrimination, though its codes are not stimulus-analogous. First, its codes discern convex and concave curves, disregarding their precise curvature. Evidence for the visual relevance of these angular categories has been obtained by Koenderink and van Doorn (1976), Hoffman and Richards (1984), Biederman (1987), and Hulleman et al (1999). Moreover, the two kinds of curves are taken as the terms of a primitive code allowing further processing. According to structural information theory (SIT) further processing implies a reduction of the primitive code on the basis of basic regularities. The resulting codes represent sets of patterns with a common regularity structure. This topic is elaborated elsewhere (van der Helm and Leeuwenberg 1991).

There are still restrictions on the size of pattern classes represented by codes of the M-system. Let us mention two restrictions. One deals with the visual ability to discern quantitative variations within convex and within concave curves (Goldmeier 1972). In line with the distinction, made by MacKay (1969), between structural dimensions of patterns and metric variations within these dimensions, we assume that the precise degree of the curvature of convex and concave angles is part of a pattern code, but at a subordinate level within each angular category. The other restriction is that codes ought to be viewpoint-dependent or image-based, otherwise they are not handedness-sensitive anyhow (Hinton and Parsons 1981; Takano 1989; Bülthoff and Edelman 1992; Tarr 1995). This restriction is implemented in the representation of α and β screws as follows: Each screw image has a 2-D component and a depth component. Thus, a screw image is neither represented as a 2-D object projection nor as a 3-D viewpoint-independent structure. Instead, the representation is partly at 2-D level and partly at 3-D level, let us say at a 2.5-D level, by means of a viewer-centred object code.

The argument for this intermediate level is the following. In handedness-recognition tasks, both perception and imagery are involved. In an initial stage shapes are perceived, and in a subsequent stage a mental picture of their rotation will be formed. According to Marr and Nishihara (1978), perception passes through three stages: a 2-D retinal image, a 2.5-D or viewer-centred object representation, and finally a 3-D or object-centred code. According to Pinker and Finke (1980) imagery may pass through these stages in the reverse order, namely from 3-D towards 2.5-D and 2-D representations. The central type of representation, being 2.5-D, comprises the visually relevant features of both the proximal and the distal aspects of objects. More direct evidence for the importance of proximal aspects of objects is given by Pani et al (1996). They showed that, if components of screws are aligned with, for instance, the viewing axis, mental rotation is facilitated.

5.2 *Rotation index*

A rotation index is a code cue which provides information about the kind of rotation which relates two object images. This information is of use at the start of an image–mirror-image discrimination test to guide mental rotation. For pairs of single screws, the codes of the H-system merely lead to a binary uncertainty, ie they do not discern backward and forward rotations. However, for chains of screws this binary uncertainty for each pair of screws leads to an explosive amount of confusions. This implies that, if mental rotation is used, innumerable rotations would have to be tested by mental rotation. This is not true for codes of the M-system.

We cannot guarantee that the rotation indices of the M-system are always accessible. This is especially uncertain in cases where components of screw objects do not coincide with the axes of the viewer (Pani et al 1996). Nor do we believe that rotation indices are regularly used by subjects, certainly not without some training. Nevertheless, the indices of the M-system are, at least in principle, accessible.

In vision studies, attempts have been made to establish a visual anchor that provides information about the minimal amount of rotation to be tested by mental rotation in the picture plane. Typically, the axis of a pattern that specifies the orientation of the pattern is taken as such an anchor. As mentioned earlier, some researchers assume that this axis can be established independently of the shape (Marr and Nishihara 1978; Hinton and Parsons 1981; Palmer 1982). The expectation is that a priori knowledge of this axis reduces the relation between reaction time and pattern orientation in case of image–mirror-image discrimination. This expectation is indeed supported by experiments (Hinton and Parsons 1981). However, Gauthier and Tarr (1997) show effects of orientation-priming that are consistent with the view that orientation and shape are not dissociable (Cooper and Shepard 1973; Rock 1973; Corballis 1988), ie that the axis is derived from the shape code and is not established separately. We agree with the latter view. Elsewhere we have shown how axes are incorporated in shape codes (Leeuwenberg and van der Helm 1991; Leeuwenberg et al 1994; van Lier et al 1997).

5.3 *Direct viewpoint effects*

This study focuses on aspects of mental rotation. Mental rotation is a hypothetical construct that is assumed to explain the specific recognition times in an image–mirror-image discrimination task (Shepard and Metzler 1971). These recognition effects, which we call 'indirect viewpoint effects', have led us to the assumption of image-based object codes. However, the same conclusion can be derived from recognition effects in more common tasks in which differences between images and mirror-images are absent or disregarded (Farah and Hammond 1988; Tarr 1995; Perret et al 1998). The latter recognition effects do not require the assumption of mental rotation. We call them 'direct-viewpoint effects'. An instance of this effect is the following: if only a few specific projections of an object are familiar, the recognition is better the more a new stimulus agrees with one of these familiar projections (Bülthoff and Edelman 1992). A question is why we do not resort to these direct-viewpoint effects in order to arrive at image-based object codes?

The answer is that codes of the M-system, derived from indirect-viewpoint effects, explain more perceptual phenomena than codes derived from direct-viewpoint effects. Our arguments are the following: direct-viewpoint effects may give rise to the assumption of *multiple-image-based* object codes (Tarr and Bülthoff 1998). Such a code might be an assembly of representations which, each, represents an object incompletely. An instance of such a code is an assembly of various stimulus-analogous representations of an object from different points of views. Each of these representations, on its own, is ambiguous, whereas only their combination may be unambiguous. Equally, an assembly of feature descriptions may constitute such a multiple-image-based object code.

Without doubt, such a code explains, besides direct-viewpoint effects, implicit handedness representation and, hence, the need for mental rotation in an image – mirror-image discrimination experiment. However, it is not clear whether this code supplies a rotation index. Its sub-representations, each, do not completely represent the object. Thus, these sub-representations do not sufficiently reveal the rotation relations between the different object views.

A code of the M-system is not a multiple-image-based object code but is a *single-image-based* object code. Indeed, this code contains information about the orientation with respect to the viewer. For instance, a screw gives rise to eight different image-based object codes (see figure 13). Yet, each of these codes completely represents the screw object in an unambiguous way. All these eight codes of the screw can be derived from each other by code rotation (see comment on correct coding under rotation). Thus, a code of the M-system can be conceived as an object-centred structural code, enriched with viewpoint information. Such an integration of object-centred and image-based coding may agree with what Tarr and Bülthoff (1998) are aiming at.

Being an image code, a code of the M-system also explains direct-viewpoint effects if no use is made of mental rotation. In this instance, it is plausible that, if only a few specific projections of an object are familiar, only their representations are highly activated. Then, it is equally plausible that recognition of a new stimulus is better the more this stimulus agrees with one of the familiar projections of the object. Another viewpoint effect occurs where differences between images and mirror-images are disregarded in recognition tasks. In contrast to the codes of the H-system, the codes of the M-system of images and of mirror-images consist of the same combinations of components. Only their order is different. This implies that, without mental rotation, the latter codes allow to conceive images and mirror-images as the same structures.

6 Conclusive remarks

To sum up, it may be said that codes that do not provide a handedness cue but rather provide a rotation index for guiding mental rotation have received most support from experiments on visual perception. This implies that most studies favour the implications as well as the assumptions of the M-system.

An implication of the M-system is that a code is an object-centred structural representation and is still view-dependent. This code combines object-centred and viewpoint-dependent aspects of objects, and there is a lot of evidence that coding models which focus on either aspect of objects are visually relevant but still insufficient (Tarr and Bülthoff 1998). For instance, the object-centred structural-coding approaches, proposed by Marr and Nishihara (1978), Biederman (1987), and Leeuwenberg and van der Helm (1991), are of use for discriminating objects of different categories but not for discriminating visually similar objects, whereas the image-based coding approaches, advocated by Bülthoff and Edelman (1992); Humphrey and Kahn (1992), and Tarr (1995), are appropriate for the latter kind and less appropriate for the first kind of discrimination (Jolicoeur 1990; Hummel 1998).

An assumption of the M-system is that perception uses a symmetric rather than an asymmetric reference frame. In other words, perception is initially only sensitive to symmetric structures and insensitive to asymmetric structures. This conclusion is also supported by direct evidence from vision research. It has been frequently reported that bilateral symmetry, especially around the vertical axis, is the most salient regularity of perception (Mach 1886/1959; Palmer and Hemenway 1978; Zimmer 1984). However, at present the jury is still out on the cause of the prominent role of symmetry. Most researchers advocate that this role is due to an external source (Pomerantz and Kubovy 1986; Wagemans et al 1993; Tyler et al 1995): symmetry is both frequent and functionally important for living organisms, hence perception has become sensitive to symmetry during evolution. Furthermore, as was stated in the introduction, differences between images and mirror-images occur less frequently and are rarely important in everyday perception. It may be less harmful to confuse images and mirror-images than to conceive them as unrelated objects.

There are also researchers who primarily reduce the visual sensitivity to symmetry to an intrinsic property of symmetry. The actual frequency of symmetry in the environment and its functional importance are usually considered by them to be irrelevant. Proponents of this view are Palmer (1983), van der Helm and Leeuwenberg (1996), and Enquist and Arak (1994). Palmer (1983) refers to symmetry as a rigid transformation having group properties. Van der Helm and Leeuwenberg (1996) refer to the 'holographic' property of symmetry. This property makes symmetry accessible from any of its substructures. Enquist and Arak (1994) assume that it is precisely the perceptual sensitivity of animals to symmetry that is decisive in partner selection, and thus accounts for the incidence of symmetrical animals in subsequent generations.

With our view, we belong to the latter camp of authors. It is our assumption that the visual insensitivity to handedness is related to the mathematical proof that handedness is indefinable (Freudenthal 1962). This relation between vision and mathematics is perhaps not very compelling. As detector, perception could have adopted a specific asymmetric structure from an external source, for instance from the rotation of the Sun in the northern hemisphere of the Earth, or from the left–right asymmetry between the brain hemispheres. Nevertheless, the suggested relation between vision and mathematics gives rise to the speculation that the visual system makes use only of intrinsically definable concepts and refuses to be sensitive to a content which, of necessity, has to be learned or adopted from an external source. Indeed, this system is not an expert system that is well-adapted to its specific surroundings. Instead, it is rather naive, but, for this very reason, also flexible and adaptive to the ever-changing environment. In our view, these are plausible properties of perception as the entry process of cognition.

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